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Target Mass Corrections for Polarized Structure Functions and New Sum Rules

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Abstract

The target mass corrections are calculated for all structure functions of neutral and charged current deep inelastic scattering in lowest order in the coupling constant. Representations of the correction to the twist–2 and twist–3 contributions are derived both in Mellin-n and x-space. The impact of the target mass corrections on the general relations between the twist–2 and twist–3 parts of the structure functions is studied and three new relations between the twist–3 contributions are derived.

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1 Introduction

Deep inelastic lepton–nucleon scattering provides one of the cleanest possibilities to study the nucleon structure at short distances. The light–cone expansion [1] proved to be one of the most powerful techniques in the Bjorken limit, $Q^2, P.q \to \infty, x = Q^2/2P.q = \mathsf{const.}$, to derive the structure of the scattering cross sections, relations between the structure functions and their scaling violations.

The early unpolarized deep inelastic scattering experiments [2] and most of the experiments, in which polarized lepton scattering off a polarized target [3, 4] has been studied so far, operated in the range of lower values of Q^2. In this domain nucleon–mass corrections cannot be neglected. The target mass effects of $O((M^2/Q^2)^k)$ form one contribution to the power corrections. Unlike the case for dynamical higher twist effects [5–7] the target mass corrections can be calculated in closed form in all orders in M^2/Q^2 for deep inelastic structure functions. Because the product of a mass factor M^k and a *genuine* twist–l operator forms an operator of twist $\tau = k + l$ the individual terms in the Taylor–expansion in M^2/Q^2 of the target mass corrections mix with the dynamical higher twist operators under renormalization. Therefore a fully consistent description would require a common treatment of both contributions. Little is known so far on the relative strength of dynamical higher twist operators and their scaling violations. Before one may try to pursue a common treatment various conceptional problems concerning higher twist operators have first to be solved. Due to this we limit the present analysis to a systematic study of the target mass corrections for all deep inelastic structure functions extending earlier investigations [8, 9].

Two methods were proposed in the literature for the evaluation of the target mass corrections. Nachtmann [10] translated the usual power–series expansion into an expansion of operators of definite spin. The problem may be solved by applying the representation–theory of the Lorentz group. A second method is due to Georgi and Politzer [8], see also [11], in which the individual nucleon mass terms are collected and resummed. In the domain of large values of x, $x \sim 1$, these methods lead to different results for the operators of lowest twist. This is the region, however, in which also the dynamical higher twist terms of any order contribute, which have to be included into the analysis in this kinematic domain as well.

We will apply the method of Ref. [8] to all polarized structure functions.[2] The contributions due to the twist–2 and twist–3 operators are studied individually. Both the corrections for the Mellin moments of the different structure functions and the inverse Mellin transform to Bjorken-x space, which can be derived analytically, are provided. As will be shown, the complete resummation of the different series in $(M^2/Q^2)^k$ is required to avoid artificially large terms in the region $x \to 1$.

A second goal of our investigation is to study the effect of the target mass corrections on the integral relations between the different polarized structure functions in lowest order in the coupling constant. As was shown in a previous analysis [15] the twist–2 contributions to the polarized structure functions are connected by three (integral) relations, the Dicus–relation [16], the Wandzura–Wilczek relation [17] and a new relation given in Ref. [15]. A general relation which holds for all spins n could also be derived for the valence part of the twist–3 contributions to the structure functions g_2 and g_3. As in Ref. [15] nucleon mass effects were disregarded, except of those implied by the kinematics in the Born cross sections, twist–3 contributions to the structure functions g_1, g_4 and g_5 were not obtained. This picture has to be regarded as partly incomplete, since nucleon mass effects have either to be accounted for thoroughly or to

[2] The target mass corrections for g_1 and g_2 were derived in [12, 13] and to the polarized Bjorken sum rule in [14] following Ref. [10].

be neglected at all. In the latter case, however, the scattering cross sections for longitudinally polarized nucleons would even not contain the structure functions g_2 and g_3 and therefore not allow to obtain two of the twist–2 relations, namely the Wandzura–Wilczek relation and the new twist–2 relation of Ref. [15]. On the other hand, a rather asymmetric picture is yet obtained for relations in the twist–3 sector. Particularly, there is no relation yet for the complete twist–3 term contained in g_2 and a potential other structure function. As will be shown below the *complete* inclusion of the target mass corrections solves this problem since all polarized structure functions obtain, besides the twist–2 contributions, by virtue of these corrections also twist–3 terms. Similar to the case of the twist–2 contributions observed before three *new* relations are implied for the twist–3 parts of the structure functions in lowest order in the coupling constant. These relations provide the possibility of a thorough test of the twist–3 structure of the nucleon.

The paper is organized as follows. In section 2 the cross sections are summarized for neutral and charged current deep inelastic scattering allowing also for contributions due to current non–conservation. In section 3 the structure of the forward Compton amplitude is derived and section 4 provides the general expressions of the operator product expansion in the presence of target mass corrections. The nucleon matrix elements are evaluated in section 5. The relation of the moments of the twist–2 and twist–3 contributions to the different polarized structure functions are given in section 6. In section 7 the x–space representations for the twist–2 and twist–3 contributions of the target mass corrections to the polarized structure functions are provided. Section 8 deals with the effect of the target mass corrections on the twist–2 relations between the polarized structure functions in the massless limit. Three new relations are derived between the twist–3 contributions of the polarized structure functions in section 9. There also other twist–3 relations are discussed. Section 10 contains the conclusions and an appendix deals with the quark–mass corrections in the case of the Wandzura–Wilczek relation.

2 The Scattering Cross Section

The differential Born cross section for polarized lepton–polarized nucleon scattering is given by

$$\frac{d^3\sigma}{dxdyd\phi} = \frac{y\alpha^2}{Q^4} \sum_i \eta_i(Q^2) L_i^{\mu\nu} W_i^{\mu\nu} \ . \tag{1}$$

Here the index i denotes the different current combinations, i.e. $i = |\gamma|^2, |\gamma Z|, |Z|^2$ for neutral and $i = |W^\pm|^2$ for charged current interactions. ϕ is the azimuthal angle of the final-state lepton, $x = Q^2/(2P.q) \equiv Q^2/(2\nu)$ and $y = P.q/k.P$ are the Bjorken variables, where $q = k - k'$ the four momentum transfer to the hadronic vertex and $Q^2 = -q^2$. P, k and k' are the proton–, initial– and final state lepton four–momenta, respectively. The factors $\eta_i(Q^2)$ denote the ratios of the corresponding propagator terms to the photon propagator squared,

$$\begin{aligned}
\eta^{|\gamma|^2}(Q^2) &= 1, \\
\eta^{|\gamma Z|}(Q^2) &= \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \frac{Q^2}{Q^2 + M_Z^2}, \\
\eta^{|Z|^2}(Q^2) &= (\eta^{|\gamma Z|})^2(Q^2), \\
\eta^{|W^\pm|^2}(Q^2) &= \frac{1}{2}\left(\frac{G_F M_W^2}{4\pi\alpha} \frac{Q^2}{Q^2 + M_W^2}\right)^2 \ .
\end{aligned} \tag{2}$$

α is the fine structure constant, G_F the Fermi constant and M_Z and M_W are the Z and W boson masses.

The leptonic tensor has the following form

$$L^i_{\mu\nu} = \sum_{\lambda'} \left[\bar{u}(k',\lambda')\gamma_\mu(g_V^{i_1} + g_A^{i_1}\gamma_5)u(k,\lambda)\right]^\dagger \bar{u}(k',\lambda')\gamma_\nu(g_V^{i_2} + g_A^{i_2}\gamma_5)u(k,\lambda) \ . \tag{3}$$

Here, λ denotes the helicity of the initial state lepton. The indices i_1 and i_2 refer to the currents forming the combinations i in Eq. (1), and the vector and axial vector couplings read

$$\begin{array}{rclrcl} g_V^\gamma &=& 1, & g_A^\gamma &=& 0 \ , \\ g_V^Z &=& \frac{1}{2} - 2Q_I \sin^2\theta_W, & g_A^Z &=& -\frac{1}{2} \ , \\ g_V^{W^-} &=& 1, & g_A^{W^-} &=& -1 \ . \end{array} \tag{4}$$

Here Q_I denotes the charge of the initial–state lepton. θ_W is the weak mixing angle.

The hadronic tensor is given by

$$W^i_{\mu\nu} = \frac{1}{4\pi}\int d^4x e^{iqx}\langle PS \mid [J_\mu^{i_1}(x)^\dagger, J_\nu^{i_2}(0)] \mid PS\rangle \ . \tag{5}$$

S denotes the four–vector of the nucleon spin with $S \cdot P = 0$. In the following we normalize $S^2 = -M^2$, where M is the nucleon mass. In the framework of the quark–parton model the currents J_μ^j are given by

$$J_\mu^j(x) = \sum_{f,f'} \bar{q}'_f(x)\gamma_\mu(g_V^{j,f} + g_A^{j,f}\gamma_5)q_f(x)U_{ff'} \ , \tag{6}$$

where $g_{V,A}^{j,f}$ are the electroweak couplings of the quark labeled by f. For charged current interactions $U_{ff'}$ denotes the Cabibbo-Kobayashi-Maskawa matrix and $g_V = 1, \ g_A = -1$, whereas for neutral current interactions $U_{ff'} = \delta_{ff'}$, $g_V^q = e_q, g_A = 0$ for γ, and

$$\begin{array}{rclrcll} g_V^q &=& \frac{1}{2} - \frac{4}{3}\sin^2\theta_W, & g_A^q &=& -\frac{1}{2}, & \text{for} \ \ q = u,c,t \\ g_V^q &=& -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W, & g_A^q &=& \frac{1}{2}, & \text{for} \ \ q = d,s,b \end{array} \tag{7}$$

for Z–boson exchange.

The hadronic tensor is constructed requiring Lorentz and time–reversal invariance. The general structure of the hadronic tensor is

$$\begin{aligned} W_{\mu\nu} = \quad & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \frac{\hat{P}_\mu\hat{P}_\nu}{P\cdot q}F_2(x,Q^2) - i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda P^\sigma}{2P\cdot q}F_3(x,Q^2) \\ & + \frac{q_\mu q_\nu}{P\cdot q}F_4(x,Q^2) + \frac{(p_\mu q_\nu + p_\nu q_\mu)}{2P\cdot q}F_5(x,Q^2) \\ & + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{P\cdot q}g_1(x,Q^2) + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(P\cdot qS^\sigma - S\cdot qP^\sigma)}{(P\cdot q)^2}g_2(x,Q^2) \\ & + \left[\frac{\hat{P}_\mu\hat{S}_\nu + \hat{S}_\mu\hat{P}_\mu}{2} - S\cdot q\frac{\hat{P}_\mu\hat{P}_\nu}{P\cdot q}\right]\frac{g_3(x,Q^2)}{P\cdot q} \\ & + S\cdot q\frac{\hat{P}_\mu\hat{P}_\nu}{(P\cdot q)^2}g_4(x,Q^2) + (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\frac{(S\cdot q)}{P\cdot q}g_5(x,Q^2), \\ & + i\varepsilon_{\mu\nu\lambda\sigma}\frac{P_\sigma S_\lambda}{P\cdot q}g_6(x,Q^2) + S\cdot q\frac{q_\mu q_\nu}{(P\cdot q)^2}g_7(x,Q^2) \\ & + \frac{(p_\mu q_\nu + q_\mu p_\nu)S\cdot q}{2(P\cdot q)^2}g_8(x,Q^2) + \frac{S_\mu q_\nu + S_\nu q_\mu}{2P\cdot q}g_9(x,Q^2) \ , \end{aligned} \tag{8}$$

where

$$\hat{P}_\mu = P_\mu - \frac{P \cdot q}{q^2} q_\mu \ , \qquad \qquad \hat{S}_\mu = S_\mu - \frac{S \cdot q}{q^2} q_\mu \ .$$

Here the current indices were suppressed. In general the hadronic tensor is determined by five unpolarized structure functions F_i and nine polarized structure functions g_i. Due to the structure of the electroweak couplings $g_{V,A}^q$ given above and due to current conservation, for photon exchange only the structure functions $F_{1,2}$ and $g_{1,2}$ contribute. For the weak currents in general all structure functions given above are present. The notation for the structure functions F_i, g_1 and g_2 is widely unique in the literature. However, different notations are used for the structure functions $g_i|_{i=3}^{9}$. We follow the definitions of Ref. [15] for the structure functions $g_i|_{i=3}^{5}$. For the current non–conserving structure functions our definitions coincide with those introduced in Ref. [18], cf. also [19].

From Eqs. (1,3) and (8) one obtains the differential scattering cross sections of a lepton with helicity λ off a polarized nucleon. For convenience we will consider two projections of the nucleon spin vector, choosing the spin direction longitudinally and transversely to the nucleon momentum. In the nucleon rest frame one has

$$\begin{aligned} S_L &= (0,0,0,M) \ , \\ S_T &= M(0, \cos\alpha, \sin\alpha, 0) \ . \end{aligned} \tag{9}$$

The polarized part of the scattering cross section for longitudinal nucleon polarization, integrated over the azimuthal angle ϕ, reads

$$\begin{aligned} \frac{d^2\sigma(\lambda, \pm S_L)}{dxdy} &= \pm 2\pi S \frac{\alpha^2}{Q^4} \sum_i C_i \eta_i(Q^2) \\ &\times \left[-2\lambda y \left(2 - y - \frac{2xyM^2}{S}\right) x g_1^i + 8\lambda \frac{yx^2M^2}{S} g_2^i + \frac{4xM^2}{S}\left(1 - y - \frac{xyM^2}{S}\right) g_3^i \right. \\ &- 2\left(1 + \frac{2xM^2}{S}\right)\left(1 - y - \frac{xyM^2}{S}\right) g_4^i - 2xy^2\left(1 + \frac{2xM^2}{S}\right) g_5^i \\ &+ \left. 4\lambda \frac{xyM^2}{S} g_6^i - 2\left(1 - y - \frac{xyM^2}{S}\right) g_9^i \right] \ . \end{aligned} \tag{10}$$

Correspondingly, for transversely polarized nucleons one obtains

$$\begin{aligned} \frac{d^3\sigma(\lambda, \pm S_T)}{dxdyd\phi} &= \pm S \frac{\alpha^2}{Q^4} \sum_i C_i \eta_i(Q^2) \\ &\times 2\sqrt{\frac{M^2}{S}} \sqrt{xy\left[1 - y - \frac{xyM^2}{S}\right]} \cos(\alpha - \phi) \left[-2\lambda y x g_1^i - 4\lambda x g_2^i \right. \\ &- \left. \frac{1}{y}\left(2 - y - \frac{2xyM^2}{S}\right) g_3^i + \frac{2}{y}\left(1 - y - \frac{xyM^2}{S}\right) g_4^i + 2xy g_5^i - 2\lambda g_6^i - g_9^i \right] \ . \end{aligned} \tag{11}$$

Here $C^\gamma = 1$, $C^{\gamma Z} = g_V + \lambda g_A$, $C^Z = (g_V + \lambda g_A)^2$, and $C^{W^\pm} = (1 \pm \lambda)$.

As well–known, the contributions of the structure functions g_2^i and g_3^i are suppressed by a factor of M^2/S in the longitudinal spin asymmetries

$$\Delta^L = d^2\sigma(\lambda, S_L) - d^2\sigma(\lambda, -S_L) \ . \tag{12}$$

However, they contribute at the same strength as the other structure functions to the transverse spin asymmetries

$$\Delta^T = d^2\sigma(\lambda, S_T) - d^2\sigma(\lambda, -S_T) \ , \tag{13}$$

which on their own behave $\propto M/\sqrt{S}$. The current non–conserving structure functions g_7^i and g_8^i do not contribute to the cross sections when the lepton masses are neglected.

3 The Forward Compton Amplitude

The forward Compton amplitude $T_{\mu\nu}$ is related to the hadronic tensor by

$$W^i_{\mu\nu} = \frac{1}{2\pi} \mathrm{Im}\ T^i_{\mu\nu} \ , \tag{14}$$

where

$$T^i_{\mu\nu} = i \int d^4x e^{iqx} \langle PS|(T J^{i_1\dagger}_\mu(x) J^{i_2}_\nu(0)|PS\rangle \ . \tag{15}$$

The Compton amplitude can be represented in terms of the amplitudes T^i_k and A^i_k, which are related to the unpolarized and polarized structure functions, as

$$\begin{aligned}
T^i_{\mu\nu} = \quad & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) T^i_1(q^2,\nu) + \frac{\hat{P}_\mu \hat{P}_\nu}{M^2} T^i_2(x,Q^2) - i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda P^\sigma}{2M^2} T^i_3(q^2,\nu) \\
& + \frac{q_\mu q_\nu}{M^2} T^i_4(q^2,\nu) + \frac{(p_\mu q_\nu + p_\nu q_\mu)}{2M^2} T^i_5(q^2,\nu) \\
& + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{M^2} A^i_1(q^2,\nu) + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(P\cdot q S^\sigma - S\cdot q P^\sigma)}{M^4} A^i_2(q^2,\nu) \\
& + \left[\frac{\hat{P}_\mu \hat{S}_\nu + \hat{S}_\mu \hat{P}_\mu}{2} - S\cdot q \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q}\right] \frac{A^i_3(q^2,\nu)}{M^2} \\
& + S\cdot q \frac{\hat{P}_\mu \hat{P}_\nu}{M^4} A^i_4(q^2,\nu) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \frac{(S\cdot q)}{M^2} A^i_5(q^2,\nu), \\
& + i\varepsilon_{\mu\nu\lambda\sigma}\frac{P_\sigma S_\lambda}{M^2} A^i_6(q^2,\nu) + S\cdot q \frac{q_\mu q_\nu}{M^4} A^i_7(q^2,\nu) \\
& + \frac{(P_\mu q_\nu + q_\mu P_\nu) S\cdot q}{2M^4} A^i_8(q^2,\nu) + \frac{S_\mu q_\nu + S_\nu q_\mu}{2M^2} A^i_9(q^2,\nu) \ .
\end{aligned} \tag{16}$$

The structure functions $F_i(x,Q^2)$ and $g_i(x,Q^2)$ are obtained from the amplitudes $T_i(q^2,\nu)$ and $A_i(q^2,\nu)$ by

$$\begin{aligned}
F_1(x,Q^2) &= \frac{1}{2\pi} \mathrm{Im}\ T_1(q^2,\nu) \ , \\
F_{2,3,4,5}(x,Q^2) &= \frac{1}{2\pi}\frac{\nu}{M^2} \mathrm{Im}\ T_{2,3,4,5}(q^2,\nu) \ ,
\end{aligned} \tag{17}$$

for the unpolarized structure functions, and

$$\begin{aligned}
g_{1,3,5,6,9}(x,Q^2) &= \frac{1}{2\pi}\frac{\nu}{M^2} \mathrm{Im} A_{1,3,5,6,9}(q^2,\nu) \ , \\
g_{2,4,7,8}(x,Q^2) &= \frac{1}{2\pi}\frac{\nu^2}{M^4} \mathrm{Im}\ A_{2,4,7,8}(q^2,\nu) \ ,
\end{aligned} \tag{18}$$

for the polarized structure functions. In the following we will consider the polarized part of $T^i_{\mu\nu}$ only.

For neutral current interactions the current operators obey

$$J_\mu^{\gamma,Z\dagger} = J_\mu^{\gamma,Z} \ . \tag{19}$$

Therefore, the crossing relation for the amplitude for $q \to -q, P \to P$ yields

$$T^i_{\mu\nu}(q^2,-\nu) = T^i_{\nu\mu}(q^2,\nu) \ . \tag{20}$$

The corresponding relations for the amplitudes $A_i^{\rm NC}(q^2,\nu)$ are

$$\begin{aligned} A^{\rm NC}_{1,3,8,9}(q^2,-\nu) &= A^{\rm NC}_{1,3,8,9}(q^2,\nu) \ , \\ A^{\rm NC}_{2,4,5,6,7}(q^2,-\nu) &= -A^{\rm NC}_{2,4,5,6,7}(q^2,\nu) \ . \end{aligned} \tag{21}$$

Furthermore, the amplitudes obey the following forward dispersion relations

$$\begin{aligned} A^{\rm NC}_{1,3,8,9}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^\infty d\nu' \frac{\nu'}{\nu'^2-\nu^2} {\rm Im}\ A^{\rm NC}_{1,3,8,9}(q^2,\nu') \ , \\ A^{\rm NC}_{2,4,5,6,7}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^\infty d\nu' \frac{\nu}{\nu'^2-\nu^2} {\rm Im}\ A^{\rm NC}_{2,4,5,6,7}(q^2,\nu') \ . \end{aligned} \tag{22}$$

For the charged current interactions it is more suitable to study the linear combination of amplitudes

$$T^\pm_{\mu\nu}(q^2,\nu) = T^{W^-}_{\mu\nu}(q^2,\nu) \pm T^{W^+}_{\mu\nu}(q^2,\nu) \ . \tag{23}$$

Due to the transformation

$$J_\mu^{W^\pm\dagger} = J_\mu^{W^\mp} \ , \tag{24}$$

the crossing relations read

$$T^\pm(q^2,-\nu) = \pm T^\pm(q^2,\nu) \ . \tag{25}$$

Correspondingly, one obtains for the combination of the amplitudes

$$A_i^\pm = A_i^{W^-} \pm A_i^{W^+} \tag{26}$$

the relations

$$\begin{aligned} A^\pm_{1,3,8,9}(q^2,-\nu) &= \pm A^\pm_{1,3,8,9}(q^2,\nu) \ , \\ A^\pm_{2,4,5,6,7}(q^2,-\nu) &= \mp A^\pm_{2,4,5,6,7}(q^2,\nu) \ . \end{aligned} \tag{27}$$

The respective dispersion relations for the amplitudes $A_i^+(q^2,\nu)$ and $A_i^-(q^2,\nu)$ are

$$\begin{aligned} A^+_{1,3,8,9}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^\infty d\nu' \frac{\nu'}{\nu'^2-\nu^2} {\rm Im} A^+_{1,3,8,9}(q^2,\nu') \ , \\ A^+_{2,4,5,6,7}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^\infty d\nu' \frac{\nu}{\nu'^2-\nu^2} {\rm Im} A^+_{2,4,5}(q^2,\nu') \ , \\ A^-_{1,3,8,9}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^\infty d\nu' \frac{\nu}{\nu'^2-\nu^2} {\rm Im} A^-_{1,3,8,9}(q^2,\nu') \ , \\ A^-_{2,4,5,6,7}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^\infty d\nu' \frac{\nu'}{\nu'^2-\nu^2} {\rm Im} A^-_{2,4,5,6,7}(q^2,\nu') \ . \end{aligned} \tag{28}$$

In the case of charged current interactions we introduce the structure function combinations

$$g_i^{\pm}(x,Q^2) = g_i^{W^-}(x,Q^2) \pm g_i^{W^+}(x,Q^2) \ . \tag{29}$$

The integral representations of the amplitudes $A_i^{\rm NC}$ and $A_i^{\pm}$ can be finally expressed by the moments of the corresponding structure functions as

$$\begin{aligned}
A_{1,3,9}^{NC,+}(q^2,\nu) &= \frac{4M^2}{\nu}\sum_{n=0,2...}\frac{1}{x^{n+1}}\int_0^1 dyy^n g_{1,3,9}^{NC,+}(y,Q^2) \ , \\
A_{2,4,7}^{NC,+}(q^2,\nu) &= \frac{4M^4}{\nu^2}\sum_{n=2,4...}\frac{1}{x^{n+1}}\int_0^1 dyy^n g_{2,4,7}^{NC,+}(y,Q^2) \ , \\
A_{5,6}^{NC,+}(q^2,\nu) &= \frac{4M^2}{\nu}\sum_{n=1,3...}\frac{1}{x^{n+1}}\int_0^1 dyy^n g_{5,6}^{NC,+}(y,Q^2) \ , \\
A_{8}^{NC,+}(q^2,\nu) &= \frac{4M^4}{\nu^2}\sum_{n=1,3...}\frac{1}{x^{n+1}}\int_0^1 dyy^n g_{8}^{NC,+}(y,Q^2) \ ,
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
A_{1,3,9}^{-}(q^2,\nu) &= \frac{4M^2}{\nu}\sum_{n=1,3...}\frac{1}{x^{n+1}}\int_0^1 dyy^n g_{1,3,9}^{-}(y,Q^2) \ , \\
A_{2,4,7}^{-}(q^2,\nu) &= \frac{4M^4}{\nu^2}\sum_{n=1,3...}\frac{1}{x^{n+1}}\int_0^1 dyy^n g_{2,4}^{-}(y,Q^2) \ , \\
A_{5,6}^{-}(q^2,\nu) &= \frac{4M^2}{\nu}\sum_{n=0,2...}\frac{1}{x^{n+1}}\int_0^1 dyy^n g_{5,6}^{-}(y,Q^2) \ , \\
A_{8}^{-}(q^2,\nu) &= \frac{4M^4}{\nu^2}\sum_{n=2,4...}\frac{1}{x^{n+1}}\int_0^1 dyy^n g_{8}^{-}(y,Q^2) \ ,
\end{aligned} \tag{31}$$

performing a Taylor expansion of Eqs. (22 , 28) and using Eqs. (18 , 29), where $x = Q^2/(2\nu)$ and the integration variable $y = Q^2/(2\nu')$.

4 Operator product expansion

The operator product expansion is one of the most general formalisms to analyze the properties of the structure functions in deep-inelastic scattering. We apply it to the T-product of two electroweak currents,

$$\widehat{T}^i_{\mu\nu} = T(J_\mu^{i_1\dagger}(x)J_\nu^{i_2}(0)) \ . \tag{32}$$

Near the light cone one obtains for neutral currents

$$\begin{aligned}
\widehat{T}^{NC}_{\mu\nu} &= \bar{q}(x)\gamma_\mu(g_{V_1}+g_{A_1}\gamma_5)S(\,x)\gamma_\nu(g_{V_2}+g_{A_a}\gamma_5)P^+q(0) \\
&+ \bar{q}(0)\gamma_\nu(g_{V_2}+g_{A_2}\gamma_5)S(\,x)\gamma_\mu(g_{V_1}+g_{A_1}\gamma_5)P^+q(x) \ ,
\end{aligned} \tag{33}$$

and for the charged current combinations

$$\widehat{T}^{\pm}_{\mu\nu} = \widehat{T}^{W_-}_{\mu\nu} \pm \widehat{T}^{W^+}_{\mu\nu} \ , \tag{34}$$

where

$$\begin{aligned}\widehat{T}^{\pm}_{\mu\nu} &= \bar{q}(x)\gamma_\mu(g_{V_1}+g_{A_1}\gamma_5)S(\,x)\gamma_\nu(g_{V_2}+g_{A_a}\gamma_5)P^{\pm}q(0)\\ &\pm\ \bar{q}(0)\gamma_\nu(g_{V_2}+g_{A_2}\gamma_5)S(\,x)\gamma_\mu(g_{V_1}+g_{A_1}\gamma_5)P^{\pm}q(x)\ . \end{aligned}\tag{35}$$

Here we used the projectors

$$P^+ = \mathbf{1}, \qquad\qquad P^- = \tau_{\mathbf{3}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{36}$$

The Fourier transforms of the expressions (33) and (35) have the following form

$$\begin{aligned} i\int d^4x e^{iqx}\widehat{T}^{NC}_{\mu\nu} &= \\ &- \int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\gamma_\mu(g_{V_1}+g_{A_1}\gamma_5)\frac{\hat{k}+\hat{q}+m_q}{(k+q)^2-m_q^2}\gamma_\nu(g_{V_2}+g_{A_2}\gamma_5)P^+U(k)\\ &- \int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\gamma_\mu(g_{V_1}+g_{A_1}\gamma_5)\frac{\hat{k}-\hat{q}+m_q}{(k-q)^2-m_q^2}\gamma_\nu(g_{V_2}+g_{A_2}\gamma_5)P^+U(k), \end{aligned}\tag{37}$$

and

$$\begin{aligned} i\int d^4x e^{iqx}\widehat{T}^{\pm}_{\mu\nu} &= \\ &- \int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\gamma_\mu(g_{V_1}+g_{A_1}\gamma_5)\frac{\hat{k}+\hat{q}+m_q}{(k+q)^2-m_q^2}\gamma_\nu(g_{V_2}+g_{A_2}\gamma_5)P^{\pm}U(k)\\ &\mp \int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\gamma_\mu(g_{V_1}+g_{A_1}\gamma_5)\frac{\hat{k}-\hat{q}+m_q}{(k-q)^2-m_q^2}\gamma_\nu(g_{V_2}+g_{A_2}\gamma_5)P^{\pm}U(k). \end{aligned}\tag{38}$$

Here m_q denotes the mass of the outgoing quark in the scattering process and

$$U(k) = \int \frac{d^4k}{(2\pi)^4}\ e^{-ikx}\psi(x)\ . \tag{39}$$

In the case of forward scattering the Fourier transform of the current product depends only on one variable.

So far our discussion was quite general. In the following we will disregard the quark mass effects. As a consequence current conservation is also obtained for weak currents and only the structure functions $F_i|_{i=1}^3$ and $g_i|_{i=1}^5$ contribute. To isolate the spin dependent part in Eqs. (37) and (38) we use the identities

$$\begin{aligned}\gamma_\mu\hat{a}\gamma_\nu &= a^\alpha[S_{\mu\alpha\nu\beta}\gamma_\beta - i\varepsilon_{\mu\alpha\nu\beta}\gamma_\beta\gamma_5]\ ,\\ S_{\mu\alpha\nu\beta} &= g_{\mu\alpha}g_{\mu\beta}+g_{\mu\beta}g_{\nu\alpha}-g_{\mu\nu}g_{\alpha\beta}\ . \end{aligned}\tag{40}$$

For the spin dependent part of $\widehat{T}^i_{\mu\nu}$ one obtains

$$\widehat{T}^+_{\mu\nu} = -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha u_+^\beta + (g_{V_1}g_{A_2}+g_{A_1}g_{V_2})S_{\mu\alpha\nu\beta}[q^\alpha u_-^\beta + u^{\alpha\beta}]\ , \tag{41}$$

$$\widehat{T}^-_{\mu\nu} = -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha v_-^\beta + (g_{V_1}g_{A_2}+g_{A_1}g_{V_2})S_{\mu\alpha\nu\beta}[q^\alpha v_+^\beta + v^{\alpha\beta}]\ , \tag{42}$$

with

$$u_{\pm}^{\beta} \;=\; -\int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\frac{\gamma_\beta\gamma_5}{(k+q)^2}P^+U(k) \;\mp\; (q\leftrightarrow -q) \;, \tag{43}$$

$$u^{\alpha\beta} \;=\; -\int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\frac{k_\alpha\gamma_\beta\gamma_5}{(k+q)^2}P^+U(k) \;-\; (q\leftrightarrow -q) \;, \tag{44}$$

$$v_{\pm}^{\beta} \;=\; -\int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\frac{\gamma_\beta\gamma_5}{(k+q)^2}P^-U(k) \;\mp\; (q\leftrightarrow -q) \;, \tag{45}$$

$$v^{\alpha\beta} \;=\; -\int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\frac{k_\alpha\gamma_\beta\gamma_5}{(k+q)^2}P^-U(k) \;-\; (q\leftrightarrow -q) \tag{46}$$

in the massless quark limit $m_q \to 0$. Expanding the denominators $(k+q)^2$ and $(k-q)^2$ in the above relations into powers of the ratio $2k.q/Q^2$ the corresponding operators can be expressed in terms of the two operators $\Theta^{\pm\beta\{\mu_1,\cdots\mu_n\}}$:

$$u_{\pm}^{\beta} \;=\; \sum_{n\ even,odd}\left(\frac{2}{Q^2}\right)^{n+1} q_{\mu_1}\ldots q_{\mu_n}\Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} \;, \tag{47}$$

$$u^{\alpha\beta} \;=\; \sum_{n\ even}\left(\frac{2}{Q^2}\right)^{n+1} q_{\mu_1}\ldots q_{\mu_n}\Theta^{+\beta\{\alpha\mu_1\cdots\mu_n\}} \;, \tag{48}$$

$$v_{\pm}^{\beta} \;=\; \sum_{n\ even,odd}\left(\frac{2}{Q^2}\right)^{n+1} q_{\mu_1}\ldots q_{\mu_n}\Theta^{\mp\beta\{\mu_1\cdots\mu_n\}} \;, \tag{49}$$

$$v^{\alpha\beta} \;=\; \sum_{n\ odd}\left(\frac{2}{Q^2}\right)^{n+1} q_{\mu_1}\ldots q_{\mu_n}\Theta^{-\beta\{\alpha\mu_1\cdots\mu_n\}} \;, \tag{50}$$

where

$$\Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} = \int \frac{d^4k}{(2\pi)^4}\overline{U}(k)\gamma_\beta\gamma_5 k_{\mu_1}\ldots k_{\mu_n}P^{\pm}U(k) \;. \tag{51}$$

Inserting the expressions for the quark operators (47–50) into the spin–dependent part of the T-product, $\widehat{T}^i_{spin}$, one obtains

$$\begin{aligned}
\widehat{T}^+_{\mu\nu,spin} \;&=\; -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha \sum_{n\ even} q^{\mu_1}\cdots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1}\Theta^{+\beta\{\mu_1\cdots\mu_n\}} \\
&+\; (g_{V_1}g_{A_2}+g_{V_2}g_{A_1})\left[-g_{\mu\nu}\sum_{n\ even} q^{\mu_1}\cdots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n}\Theta^{+\mu_1\{\mu_2\cdots\mu_n\}}\right. \\
&+\; \sum_{n\ odd} q^{\mu_1}\cdots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1} + \left(q_\mu\Theta^{+\nu\{\mu_1\cdots\mu_n\}}+q_\nu\Theta^{+\mu\{\mu_1\cdots\mu_n\}}\right) \\
&+\; \left.\sum_{n\ even} q^{\mu_1}\cdots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1}\left(\Theta^{+\mu\{\nu\mu_1\cdots\mu_n\}}+\Theta^{+\nu\{\mu\mu_1\cdots\mu_n\}}\right)\right] \;,
\end{aligned} \tag{52}$$

and

$$\begin{aligned}
\widehat{T}^-_{\mu\nu,spin} \;&=\; -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha \sum_{n\ odd} q^{\mu_1}\cdots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1}\Theta^{-\beta\{\mu_1\cdots\mu_n\}} \\
&+\; (g_{V_1}g_{A_2}+g_{V_2}g_{A_1})\left[-g_{\mu\nu}\sum_{n\ odd} q^{\mu_1}\cdots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n}\Theta^{-\mu_1\{\mu_2\cdots\mu_n\}}\right.
\end{aligned}$$

$$+ \sum_{n\ even} q^{\mu_1} \cdots q^{\mu_n} \left(\frac{2}{Q^2}\right)^{n+1} \left(q_\mu \Theta^{-\nu\{\mu_1\cdots\mu_n\}} + q_\nu \Theta^{-\mu\{\mu_1\cdots\mu_n\}}\right)$$
$$\left. + \sum_{n\ odd} q^{\mu_1} \cdots q^{\mu_n} \left(\frac{2}{Q^2}\right)^{n+1} \left(\Theta^{-\mu\{\nu\mu_1\cdots\mu_n\}} + \Theta^{-\nu\{\mu\mu_1\cdots\mu_n\}}\right)\right] . \quad (53)$$

The operators $\Theta^{\pm\beta\{\mu_1\cdots\mu_n\}}$ can be decomposed into a symmetric part, Θ_S, and a remainder, Θ_R, respectively

$$\Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} = \Theta_S^{\pm\beta\{\mu_1\cdots\mu_n\}} + \Theta_R^{\pm\beta\{\mu_1]\cdots\mu_n\}} , \quad (54)$$

where

$$\Theta_S^{\pm\beta\{\mu_1\cdots\mu_n\}} = \frac{1}{n+1}\left[\Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} + \Theta^{\pm\mu_1\{\beta\cdots\mu_n\}} + \ldots + \Theta^{\pm\mu_n\{\mu_1\cdots\beta\}}\right] , \quad (55)$$
$$\Theta_R^{\pm\beta\{\mu_1\cdots\mu_n\}} = \frac{1}{n+1}\left[\Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} - \Theta^{\pm\mu_1\{\beta\cdots\mu_n\}} + \Theta^{\pm\beta\{\mu_1\mu_2\cdots\mu_n\}} - \Theta^{\pm\mu_2\{\mu_1\beta\cdots\mu_n\}} + \ldots\right] . \quad (56)$$

In the massless quark limit both operators Θ_S and Θ_R are traceless. To find the target mass dependence of the structure functions we have to construct the traceless nucleon matrix elements of these operators [8].

The rank-n symmetric, traceless tensor constructed from n nucleon momenta P_{μ_i} $(P^2 = M^2)$ has the form

$$\Pi^{\mu_1\cdots\mu_n}(P) = \sum_{j=0}^{[n/2]} \frac{(-1)^j}{2^j} \frac{(n-j)!}{n!} \underbrace{g\cdots g}_{j} \underbrace{P\cdots P}_{n-2j} M^{2j} . \quad (57)$$

Here, $[n/2] = n/2$ for even n and $[n/2] = (n-1)/2$ for odd n. The sum contains j metric tensors $g_{\mu_i\nu_k}$ with indices chosen in the set $\{\mu_1, \ldots, \mu_n\}$ in all possible combinations. The remaining $n-2j$ indices are carried by the momenta P_{μ_l}. The sum contains $n!/[2^j j!(n-2j)!]$ terms.

For the representation of the Θ-operators traceless and symmetric tensors have to be constructed from one nucleon spin vector S and n momentum vectors P. These tensors have the form [20]

$$\begin{aligned} M_1^{\mu_1\cdots\mu_n}(P,S) &\equiv \{S^{\mu_1}P^{\mu_2}\ldots P^{\mu_n}\} \\ &= \frac{1}{n}\sum_i S^{\mu_i}\Pi^{\mu_1\cdots[\mu_i]\cdots\mu_n}(P) - \frac{1}{n^2}\sum_{i<j} g_{\mu_i\mu_j} S_\alpha \Pi^{\mu_1\cdots[\mu_i]\cdots[\mu_j]\cdots\mu_n}(P) , \end{aligned} \quad (58)$$

where the indices in brackets $[\mu_i]$ indicate that the corresponding superscript is to be removed from the sequence. The coefficient of the second term in the above expression is chosen such that the trace vanishes identically. A more compact expression for these tensors was obtained in Ref. [9],

$$M_1^{\mu_1\cdots\mu_n}(P,S) = \{S^{\mu_1}P^{\mu_2}\ldots P^{\mu_n}\} = \frac{1}{n}\sum_{j=0}^{[(n-1)/2]} \frac{(-1)^j}{2^j}\frac{(n-j)!}{n!} \underbrace{g\cdots g}_{j} \underbrace{[SP\cdots P]_S}_{n-2j} M^{2j} . \quad (59)$$

Again the sum contains j times the metric tensors $g_{\mu_i\mu_k}$. The remaining $n-2j$ indices are symmetrized in the product $[SP\ldots P]_S$.

5 The Nucleon Matrix Elements

We calculate now the nucleon matrix element of the operators derived in the preceding section. The expectation value of the symmetric operators Θ_S is given by

$$\langle PS|\Theta_S^{\pm\beta\{\mu_1\cdots\mu_n\}}|PS\rangle \quad = \quad a_n^{\pm} M_1^{\beta\mu_1\cdots\mu_n} \ . \tag{60}$$

Correspondingly, for the operators Θ_R one obtains, cf. also [9],

$$\langle PS|\Theta_R^{\pm\beta\{\mu_1\cdots\mu_n\}}|PS\rangle = \frac{d_n^{\pm}}{n+1}\Big[\quad M_2^{\beta\{\mu_1\mu_2\cdots\mu_n\}} - M_2^{\mu_1\{\beta\mu_2\cdots\mu_n\}} + \ldots \\ M_2^{\beta\{\mu_1\mu_2\cdots\mu_n\}} - M_2^{\mu_n\{\mu_1\mu_2\cdots\beta\}} \quad \Big] \ , \tag{61}$$

where

$$M_2^{\beta\mu_1\cdots\mu_n}(P,S) = \frac{n+2}{n+1} S^\beta \Pi^{\mu_1\cdots\mu_n}(P) + \frac{n}{n+1} P^\beta M_1^{\mu_1\cdots\mu_n}(P,S) \ . \tag{62}$$

In the massless nucleon limit the known results, cf. Ref. [15] , are reproduced,

$$\langle PS|\Theta_S^{\pm\beta\{\mu_1\ldots\mu_n\}}|PS\rangle \quad = \quad \frac{a_n^{\pm}}{n+1}\left[S^\beta P^{\mu_1} P^{\mu_2}\ldots P^{\mu_n} + S^{\mu 1} P^\beta P^{\mu_2}\ldots P^{\mu_n} + \ldots\right] \ , \tag{63}$$

$$\begin{aligned} \langle PS|\Theta_R^{\pm\beta\{\mu_1]\ldots\mu_n\}}|PS\rangle \quad = \quad & \frac{d_n^{\pm}}{n+1}\Big[\left(S^\beta P^{\mu_1} - S^{\mu_1} P^\beta\right) P^{\mu_2}\ldots P^{\mu_n} \\ & + \left(S^\beta P^{\mu_2} - S^{\mu_2} P^\beta\right) P^{\mu_1} P^{\mu_3}\ldots P^{\mu_n} \\ & + \ldots + \left(S^\beta P^{\mu_n} - S^{\mu_n} P^\beta\right) P^{\mu_1} P^{\mu_3}\ldots P^{\mu_{n-1}}\Big] \ . \end{aligned} \tag{64}$$

$a_n^{\pm}$ and $d_n^{\pm}$ denote the twist–2 and twist–3 operator matrix elements in the case of neutral current $(+, NC)$ and charged current interactions $(\pm)$ of moment index n. These are non–perturbative quantities and are independent of the nucleon mass. All information about the target mass corrections is contained in the tensors $M_1^{\beta\mu_1\ldots\mu_n}$ and $M_2^{\beta\mu_1\ldots\mu_n}$ etc.

Let us consider first the nucleon matrix element of the first term of the operator $\widehat{T}^{+,1}_{\mu\nu,\,spin}$, Eq. (52),

$$\widehat{T}^{+,1}_{\mu\nu,spin} \quad = \quad -i(g_{V_1}g_{V_2} + g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha \sum_{n\ even} q^{\mu_1}\cdots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1}\Theta^{+\beta\{\mu_1\cdots\mu_n\}}. \tag{65}$$

to derive the structure of the target mass correction for forward Compton scattering. We, furthermore, consider only the symmetric part of the operator Θ.

$$\begin{aligned} \langle PS|\widehat{T}^{+,1}_{\mu\nu,spin}|PS\rangle \quad = \quad & -i(g_{V_1}g_{V_2} + g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha \\ \times \quad & \Bigg\{ \sum_{n\ even}\sum_{j=0}^{[n/2]} \frac{1}{x^{(n-2j+1)}}\left(\frac{M^2}{Q^2}\right)^j \frac{S^\beta}{\nu}\frac{(n-j+1)!}{j!(n-2j)!}\frac{a_n^+}{(n+1)^2} \\ & + \sum_{n\ even}\sum_{j=0}^{[n/2]} \frac{1}{x^{(n-2j+1)}}\left(\frac{M^2}{Q^2}\right)^j \frac{P^\beta(S\cdot q)}{\nu^2}\frac{(n-j+1)!}{j!(n-2j-1)!}\frac{a_n^+}{(n+1)^2}\Bigg\} \ . \end{aligned} \tag{66}$$

Shifting the summation index from n to $n = n + 2j$ both sums become independent and one obtains

$$
\begin{aligned}
\langle PS|\widehat{T}^{+,1}_{\mu\nu,spin}|PS\rangle &= -i(g_{V_1}g_{V-2} + g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha \\
&\times \Bigg\{ \sum_{n\ even}\sum_{j=0}^{\infty} \frac{1}{x^{(n+1)}}\left(\frac{M^2}{Q^2}\right)^j \frac{S^\beta}{\nu}\frac{(n+j+1)!}{j!\,n!}\frac{a^+_{n+2j}}{(n+2j+1)^2} \\
&\quad + \sum_{n\ even}\sum_{j=0}^{\infty} \frac{1}{x^{(n+1)}}\left(\frac{M^2}{Q^2}\right)^j \frac{P^\beta(S\cdot q)}{\nu^2}\frac{(n+j+1)!}{j!(n-1)!}\frac{a^+_{n+2j}}{(n+2j+1)^2}\Bigg\}\,. \qquad (67)
\end{aligned}
$$

Carrying out the same steps also for the other parts of the expressions (52) and (53) and rearranging the tensor structures according to the structure functions one obtains for the twist–2 part of the forward Compton amplitude the relations

$$
\begin{aligned}
\langle PS|\widehat{T}^{NC,+,\ \tau=2}_{\mu\nu,spin}|PS\rangle &= -i(g_{V_1}g_{V_2} + g_{A_1}g_{A_2})\Bigg\{\varepsilon_{\mu\alpha\nu\beta}\frac{q^\alpha S^\beta}{\nu}\sum_{n\ even}\frac{1}{x^{n+1}}(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)\ a^+_{n+2j} \\
&- \varepsilon_{\mu\alpha\nu\beta}\frac{q^\alpha P^\beta(S\cdot q) - q^\alpha S^\beta(P\cdot q)}{\nu^2}\sum_{n\ even}\frac{1}{x^{n+1}}n\sum_{j=0}^{\infty}\mathcal{C}(n,j)\ a^+_{n+2j}\Bigg\} \\
&+ (g_{V_1}g_{A_2} + g_{V_2}g_{A_1})\Bigg\{\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{S\cdot q}{\nu} \\
&\quad \sum_{n\ odd}\frac{1}{x^{n+1}}\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a^+_{n+2j}}{n+2j}[(n+2j+1)(n+2j)+2j] \\
&+ \left[\frac{\hat{S}^\mu\hat{P}^\nu + \hat{P}^\mu\hat{S}^\nu}{2} - \frac{\hat{P}^\mu\hat{P}^\nu}{\nu}(S\cdot q)\right]\frac{4}{\nu}\sum_{n\ odd}\frac{1}{x^n}n\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a^+_{n+2j}}{n+2j} \\
&+ \frac{\hat{P}^\mu\hat{P}^\nu}{\nu}(S\cdot q)\frac{2}{\nu}\sum_{n\ odd}\frac{1}{x^n}n(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a^+_{n+2j}}{n+2j}\Bigg\}\,, \qquad (68)
\end{aligned}
$$

and

$$
\begin{aligned}
\langle PS|\widehat{T}^{-,\ \tau=2}_{\mu\nu,spin}|PS\rangle &= -i(g_{V_1}g_{V_2} + g_{A_1}g_{A_2})\Bigg\{\varepsilon_{\mu\alpha\nu\beta}\frac{q^\alpha S^\beta}{\nu}\sum_{n\ odd}\frac{1}{x^{n+1}}(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)\ a^-_{n+2j} \\
&- \varepsilon_{\mu\alpha\nu\beta}\frac{q^\alpha P^\beta(S\cdot q) - q^\alpha S^\beta(P\cdot q)}{\nu^2}\sum_{n\ odd}\frac{1}{x^{n+1}}n\sum_{j=0}^{\infty}\mathcal{C}(n,j)\ a^-_{n+2j}\Bigg\} \\
&\quad + (g_{V_1}g_{A_2} + g_{V_2}g_{A_1})\Bigg\{(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\frac{S\cdot q}{\nu} \\
&\quad \sum_{n\ even}\frac{1}{x^{n+1}}\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a^-_{n+2j}}{n+2j}[(n+2j+1)(n+2j)+2j] \\
&+ \left[\frac{\hat{S}^\mu\hat{P}^\nu + \hat{P}^\mu\hat{S}^\nu}{2} - \frac{\hat{P}^\mu\hat{P}^\nu}{\nu}(S\cdot q)\right]\frac{4}{\nu}\sum_{n\ even}\frac{1}{x^n}n\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a^-_{n+2j}}{n+2j} \\
&+ \frac{\hat{P}^\mu\hat{P}^\nu}{\nu}(S\cdot q)\frac{2}{\nu}\sum_{n\ even}\frac{1}{x^n}n(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a^-_{n+2j}}{n+2j}\Bigg\}\,. \qquad (69)
\end{aligned}
$$

Correspondingly, the twist–3 contributions read

$$
\langle PS|\widehat{T}^{NC,+,\ \tau=3}_{\mu\nu,spin}|PS\rangle =
$$

$$
\begin{aligned}
&-i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\Big\{\varepsilon_{\mu\alpha\nu\beta}\frac{q^\alpha S^\beta}{\nu}\sum_{n\ even}\frac{1}{x^{n+1}}4\sum_{j=0}^{\infty}\mathcal{C}(n,j)j\ d^+_{n+2j}\\
&+\varepsilon_{\mu\alpha\nu\beta}\frac{q^\alpha P^\beta(S\cdot q)-q^\alpha S^\beta(P\cdot q)}{\nu^2}\\
&\qquad\qquad\times\sum_{n\ even}\frac{1}{x^{n+1}}(n+1)\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)(n+2j)\ d^+_{n+2j}\Big\}\\
&-(g_{V_1}g_{A_2}+g_{V_2}g_{A_1})\Big\{(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2})\frac{S\cdot q}{\nu}\sum_{n\ odd}\frac{1}{x^{n+1}}4\sum_{j=0}^{\infty}\mathcal{C}(n,j)j\ \frac{d^+_{n+2j}}{(n+2j)}\\
&-\Big(\frac{\hat{S}^\mu\hat{P}^\nu+\hat{P}^\mu\hat{S}^\nu}{2}-\frac{\hat{P}^\mu\hat{P}^\nu}{\nu}(S\cdot q)\Big)\frac{2}{\nu}\sum_{n\ odd}\frac{1}{x^n}\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)d^+_{n+2j}\\
&\qquad\qquad\qquad\qquad\times\left[(n-1)(n+1)+4j(n+2j+1)\right]\\
&-\frac{\hat{P}^\mu\hat{P}^\nu}{\nu}(S\cdot q)\frac{8}{\nu}\sum_{n\ odd}\frac{1}{x^n}\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)(n+j+1)j\ d^+_{n+2j}\Big\}\ , \qquad (70)
\end{aligned}
$$

and

$$
\begin{aligned}
\langle PS|\widehat{T}^{-,\ \tau=3}_{\mu\nu,spin}|PS\rangle &= -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\Big\{\varepsilon_{\mu\alpha\nu\beta}\frac{q^\alpha S^\beta}{\nu}\sum_{n\ odd}\frac{1}{x^{n+1}}4\sum_{j=0}^{\infty}\mathcal{C}(n,j)j\ d^+_{n+2j}\\
&+\ \varepsilon_{\mu\alpha\nu\beta}\frac{q^\alpha P^\beta(S\cdot q)-q^\alpha S^\beta(P\cdot q)}{\nu^2}\\
&\qquad\qquad\sum_{n\ odd}\frac{1}{x^{n+1}}(n+1)\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)(n+2j)\ d^+_{n+2j}\Big\}\\
&-\ (g_{V_1}g_{A_2}+g_{V_2}g_{A_1})\Big\{(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2})\frac{S\cdot q}{\nu}\sum_{n\ even}\frac{1}{x^{n+1}}4\sum_{j=0}^{\infty}\mathcal{C}(n,j)j\ \frac{d^+_{n+2j}}{(n+2j)}\\
&-\ \Big(\frac{\hat{S}^\mu\hat{P}^\nu+\hat{P}^\mu\hat{S}^\nu}{2}-\frac{\hat{P}^\mu\hat{P}^\nu}{\nu}(S\cdot q)\Big)\frac{2}{\nu}\sum_{n\ even}\frac{1}{x^n}\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)d^+_{n+2j}\\
&\qquad\qquad\qquad\qquad\times\left[(n-1)(n+1)+4j(n+2j+1)\right]\\
&-\ \frac{\hat{P}^\mu\hat{P}^\nu}{\nu}(S\cdot q)\frac{8}{\nu}\sum_{n\ even}\frac{1}{x^n}\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)(n+j+1)j\ d^+_{n+2j}\Big\}\ , \qquad (71)
\end{aligned}
$$

where the combinatoric factors $\mathcal{C}(n,j)$ and $\mathcal{C}_1(n,j)$ are given by

$$
\begin{aligned}
\mathcal{C}(n,j) &= \left(\frac{M^2}{Q^2}\right)^j\frac{(n+j+1)!}{j!n!(n+2j+1)^2}\ , \qquad (72)\\
\mathcal{C}_1(n,j) &= \left(\frac{M^2}{Q^2}\right)^j\frac{(n+j+1)!}{j!n!(n+2j+1)(n+2j+2)^2}\ . \qquad (73)
\end{aligned}
$$

In the presence of target mass effects all polarized structure functions receive twist–3 contributions. Implicitly this effect was contained in Ref. [13] already in the case of the structure function g_1.

6 Expressions for the Moments of Structure Functions

To obtain the moments of the twist–2 and the twist–3 parts of the individual polarized structure functions we first consider the target mass corrections to the single amplitudes $A_{i,\tau=2,3}^{NC,+,-}$. The twist–2 terms are :

$$\begin{aligned}
A_{1\ \tau=2}^{NC,+}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ even}\frac{((g_V^q)^2+(g_A^q)^2)}{x^{n+1}}(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)a_{n+2j}^{+q}\ ,\\
A_{2\ \tau=2}^{NC,+}(q^2,\nu) &= -\frac{M^2}{\nu}\sum_{n\ even}\frac{((g_V^q)^2+(g_A^q)^2)}{x^{n+1}}n\sum_{j=0}^{\infty}\mathcal{C}(n,j)a_{n+2j}^{+q}\ ,\\
A_{3\ \tau=2}^{NC,+}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ odd}\frac{8g_V^qg_A^q}{x^n}n\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a_{n+2j}^{+q}}{n+2j}\ ,\\
A_{4\ \tau=2}^{NC,+}(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ odd}\frac{4g_V^qg_A^q}{x^n}n(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a_{n+2j}^{+q}}{n+2j}\ ,\\
A_{5\ \tau=2}^{NC,+}(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ odd}\frac{2g_V^qg_A^q}{x^{n+1}}\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a_{n+2j}^{+q}}{n+2j}[(n+2j+1)(n+2j)+2j]\ ,
\end{aligned}\tag{74}$$

and

$$\begin{aligned}
A_{1\ \tau=2}^{-}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ odd}\frac{((g_V^q)^2+(g_A^q)^2)}{x^{n+1}}(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)a_{n+2j}^{-q}\ ,\\
A_{2\ \tau=2}^{-}(q^2,\nu) &= -\frac{M^2}{\nu}\sum_{n\ odd}\frac{((g_V^q)^2+(g_A^q)^2)}{x^{n+1}}n\sum_{j=0}^{\infty}\mathcal{C}(n,j)a_{n+2j}^{-q}\ ,\\
A_{3\ \tau=2}^{-}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ even}\frac{8g_V^qg_A^q}{x^n}n\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a_{n+2j}^{-q}}{n+2j}\ ,\\
A_{4\ \tau=2}^{-}(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ even}\frac{4g_V^qg_A^q}{x^n}n(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a_{n+2j}^{-q}}{n+2j}\ ,\\
A_{5\ \tau=2}^{-}(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ even}\frac{2g_V^qg_A^q}{x^{n+1}}\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a_{n+2j}^{-q}}{n+2j}[(n+2j+1)(n+2j)+2j]\ .
\end{aligned}\tag{75}$$

The twist–3 terms read :

$$\begin{aligned}
A_{1\ \tau=3}^{NC,+}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ even}\frac{((g_V^q)^2+(g_A^q)^2)}{x^{n+1}}4\sum_{j=0}^{\infty}\mathcal{C}(n,j)jd_{n+2j}^{+q}\ ,\\
A_{2\ \tau=3}^{NC,+}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ even}\frac{((g_V^q)^2+(g_A^q)^2)}{x^{n+1}}\sum_{j=0}^{\infty}(n+1)\mathcal{C}_1(n-1,j)(n+2j)d_{n+2j}^{+q}\ ,\\
A_{3\ \tau=3}^{NC,+}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ odd}\frac{4g_V^qg_A^q}{x^n}\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)\left[(n-1)(n+1)+4j(n+2j+1)\right]d_{n+2j}^{+q}\ ,\\
A_{4\ \tau=3}^{NC,+}(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ odd}\frac{16g_V^qg_A^q}{x^n}n(n+1)\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)(n+j+1)jd_{n+2j}^{+q}\ ,\\
A_{5\ \tau=3}^{NC,+}(q^2,\nu) &= -\frac{M^4}{\nu^2}\sum_{n\ odd}\frac{8g_V^qg_A^q}{x^{n+1}}\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{d_{n+2j}^{+q}}{n+2j}\ ,
\end{aligned}\tag{76}$$

and

$$
\begin{aligned}
A^{-}_{1\ \tau=3}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ odd}\frac{((g_V^q)^2+(g_A^q)^2)}{x^{n+1}}4\sum_{j=0}^{\infty}\mathcal{C}(n,j)jd^{+q}_{n+2j}\ ,\\
A^{-}_{2\ \tau=3}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ odd}\frac{((g_V^q)^2+(g_A^q)^2)}{x^{n+1}}\sum_{j=0}^{\infty}(n+1)\mathcal{C}_1(n-1,j)(n+2j)d^{+q}_{n+2j}\ ,\\
A^{-}_{3\ \tau=3}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ even}\frac{4g_V^q g_A^q}{x^{n}}\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)\left[(n-1)(n+1)+4j(n+2j+1)\right]d^{+q}_{n+2j}\ ,\\
A^{-}_{4\ \tau=3}(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ even}\frac{16g_V^q g_A^q}{x^{n}}n(n+1)\sum_{j=0}^{\infty}\mathcal{C}_1(n-1,j)(n+j+1)jd^{+q}_{n+2j}\ ,\\
A^{-}_{5\ \tau=3}(q^2,\nu) &= -\frac{M^4}{\nu^2}\sum_{n\ even}\frac{8g_V^q g_A^q}{x^{n+1}}\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{d^{+q}_{n+2j}}{n+2j}\ . \qquad (77)
\end{aligned}
$$

From the representations Eqs. (30,31) the following relations between the moments of the twist–2 and twist–3 parts of the structure functions $g_i^{\mathrm{NC},\pm}(x,Q^2)$ and the operator matrix elements $a_n^{\pm,q}$ and $d_n^{\pm,q}$ are obtained. The twist–2 contributions are

$$\int_0^1 dx\ x^n g^{NC,+}_{1\ \tau=2}(x,Q^2) = \sum_q\frac{(g_V^q)^2+(g_A^q)^2}{4}(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)a^{+q}_{n+2j}\ , \quad n=0,2\ ... \qquad (78)$$

$$\int_0^1 dx\ x^n g^{NC,+}_{2\ \tau=2}(x,Q^2) = -\sum_q\frac{(g_V^q)^2+(g_A^q)^2}{4}\ n\ \sum_{j=0}^{\infty}\mathcal{C}(n,j)a^{+q}_{n+2j}\ , \quad n=2,4\ ... \qquad (79)$$

$$\int_0^1 dx\ x^n g^{NC,+}_{3\ \tau=2}(x,Q^2) = \sum_q 2g_V^q g_A^q(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n+1,j)\frac{a^{+q}_{n+2j+1}}{(n+2j+1)}\ , \quad n=0,2\ ... \qquad (80)$$

$$\int_0^1 dx\ x^n g^{NC,+}_{4\ \tau=2}(x,Q^2) = \sum_q\ g_V^q g_A^q(n+1)(n+2)\sum_{j=0}^{\infty}\mathcal{C}(n+1,j)\frac{a^{+q}_{n+2j+1}}{(n+2j+1)}\ , \quad n=2,4\ ... \qquad (81)$$

$$\int_0^1 dx\ x^n g^{NC,+}_{5\tau=2}(x,Q^2) = \sum_q\frac{1}{2}g_V^q g_A^q\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a^{+q}_{n+2j}}{(n+2j)} \times[(n+2j+1)(n+2j)+2j], \quad n=1,3\ ..., \qquad (82)$$

and

$$\int_0^1 dx\ x^n g^{-}_{1\tau=2}(x,Q^2) = \sum_q\frac{(g_V^q)^2+(g_A^q)^2}{4}(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n,j)a^{-q}_{n+2j}\ , \quad n=1,3\ ... \qquad (83)$$

$$\int_0^1 dx\ x^n g^{-}_{2\tau=2}(x,Q^2) = -\sum_q\frac{(g_V^q)^2+(g_A^q)^2}{4}\ n\ \sum_{j=0}^{\infty}\mathcal{C}(n,j)a^{-q}_{n+2j}\ , \quad n=1,3\ ... \qquad (84)$$

$$\int_0^1 dx\ x^n g^{-}_{3\tau=2}(x,Q^2) = \sum_q 2g_V^q g_A^q(n+1)\sum_{j=0}^{\infty}\mathcal{C}(n+1,j)\frac{a^{-q}_{n+2j+1}}{(n+2j+1)}\ , \quad n=1,3\ ... \qquad (85)$$

$$\int_0^1 dx\ x^n g^{-}_{4\tau=2}(x,Q^2) = \sum_q\ g_V^q g_A^q(n+1)(n+2)\sum_{j=0}^{\infty}\mathcal{C}(n+1,j)\frac{a^{-q}_{n+2j+1}}{(n+2j+1)}\ , \quad n=1,3\ ... \qquad (86)$$

$$\int_0^1 dx\ x^n g^{-}_{5\tau=2}(x,Q^2) = \sum_q\frac{1}{2}g_V^q g_A^q\sum_{j=0}^{\infty}\mathcal{C}(n,j)\frac{a^{-q}_{n+2j}}{(n+2j)}$$

$$\times[(n+2j+1)(n+2j)+2j], \quad n = 0, 2 \ldots \tag{87}$$

For the twist–3 contributions to the structure functions the moments are given by

$$\int_0^1 dx\ x^n g_{1\ \tau=3}^{NC,+}(x,Q^2) = \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4} 4 \sum_{j=0}^{\infty} \mathcal{C}(n,j) j d_{n+2j}^{+q}\ , \qquad n = 0, 2 \ldots \tag{88}$$

$$\int_0^1 dx\ x^n g_{2\ \tau=3}^{NC,+}(x,Q^2) = \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4} \sum_{j=0}^{\infty} \mathcal{C}_1(n-1,j)(n+2j)(n+1) d_{n+2j}^{+q}\ , \qquad n = 2, 4 \ldots \tag{89}$$

$$\int_0^1 dx\ x^n g_{3\ \tau=3}^{NC,+}(x,Q^2) = \sum_q g_V^q g_A^q \sum_{j=0}^{\infty} \mathcal{C}_1(n,j) d_{n+2j+1}[(n+2j)(n+2j+2)+4j]\ , \qquad n = 0, 2 \ldots \tag{90}$$

$$\int_0^1 dx\ x^n g_{4\tau=3}^{NC,+}(x,Q^2) = 4 \sum_q g_V^q g_A^q \sum_{j=0}^{\infty} \mathcal{C}_1(n,j)(n+j+2) j d_{n+2j+1}\ , \qquad n = 2, 4 \ldots \tag{91}$$

$$\int_0^1 dx\ x^n g_{5\tau=3}^{NC,+}(x,Q^2) = -2 \sum_q g_V^q g_A^q \sum_{j=0}^{\infty} \mathcal{C}(n,j) j \frac{d_{n+2j}}{n+2j}\ , \qquad n = 1, 3 \ldots \tag{92}$$

and

$$\int_0^1 dx\ x^n g_{1\ \tau=3}^{-}(x,Q^2) = \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4} 4 \sum_{j=0}^{\infty} \mathcal{C}(n,j) j d_{n+2j}^{-q}\ , \qquad n = 1, 3 \ldots \tag{93}$$

$$\int_0^1 dx\ x^n g_{2\ \tau=3}^{-}(x,Q^2) = \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4} \sum_{j=0}^{\infty} \mathcal{C}_1(n-1,j)(n+2j)(n+1) d_{n+2j}^{-q}\ , \qquad n = 1, 3 \ldots \tag{94}$$

$$\int_0^1 dx\ x^n g_{3\ \tau=3}^{-}(x,Q^2) = \sum_q g_V^q g_A^q \sum_{j=0}^{\infty} \mathcal{C}_1(n,j) d_{n+2j+1}[(n+2j)(n+2j+2)+4j]\ , \qquad n = 1, 3 \ldots \tag{95}$$

$$\int_0^1 dx\ x^n g_{4\ \tau=3}^{-}(x,Q^2) = 4 \sum_q g_V^q g_A^q \sum_{j=0}^{\infty} \mathcal{C}_1(n,j)(n+j+2) j d_{n+2j+1}\ , \qquad n = 1, 3 \tag{96}$$

$$\int_0^1 dx\ x^n g_{5\ \tau=3}^{-}(x,Q^2) = -2 \sum_q g_V^q g_A^q \sum_{j=0}^{\infty} \mathcal{C}(n,j) j \frac{d_{n+2j}}{n+2j}\ , \qquad n = 0, 2 \ldots \tag{97}$$

Eqs. (78–97) can be used directly in structure function analyses based on integer moments. Unlike the massless case the operator expectation values $a_n^{\pm}$ and $d_n^{\pm}$ do not decouple for a given spin n in this representation. As will be shown in the subsequent section, however, the infinite sums in Eqs. (78–97) can be related to integrals over one–dimensional partition functions via an analytic continuation from the integers n to the complex n-plane. Carlson's theorem [21] assures the uniqueness of the analytic continuation and the inverse Mellin transform.

7 The Inverse Mellin Transform

In many practical applications, as the analysis of experimental data, the expressions for the moments of the twist–2 and twist–3 contributions to the deep inelastic scattering structure

functions at integer values of n are less suited than the corresponding x–space expressions. For this purpose we perform the inverse Mellin transform of the results derived in the last section and provide integral representations which can be directly applied to the measured structure functions, which are given as distributions in x and Q^2, to unfold the target mass corrections.

We apply a procedure analogous to that used by Georgi and Politzer in Ref. [8] in the case of unpolarized structure functions. We summarize the different steps for the case of the structure functions $g_1^{\pm}(x, Q^2)$. From Eqs. (78,83) one obtains

$$\begin{aligned} g_{1\ \tau=2}^{\pm}(x,Q^2) &= \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4} \\ &\times \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn\ x^{-(n+1)}(n+1)\sum_{j=0}^{\infty}\left(\frac{M^2}{Q^2}\right)^j \frac{(n+j+1)!}{j!n!}\frac{a_{n+2j}^{\pm,q}}{(n+2j+1)^2}\ . \end{aligned} \tag{98}$$

The operator expectation values $a_n^{\pm}$ are the moments of distribution functions $F^{\pm q}(x)$, which are related to the polarized parton densities in the massless limit, $\Delta q(x) \pm \Delta \overline{q}(x)$, cf. e.g. [15],

$$a_n^{\pm,q} = \int_0^1 dy y^n F^{\pm q}(y)\ . \tag{99}$$

Hence,

$$\frac{a_{n+2j}^{\pm}}{(n+2j+1)^2} = \int_0^1 dy y^{n+2j} G_1^{\pm}(y)\ , \tag{100}$$

where

$$G_1^{\pm}(y) = \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4}\int_y^1 \frac{dy_1}{y_1}\int_{y_1}^1 \frac{dy_2}{y_2} F^{\pm q}(y_2)\ . \tag{101}$$

Inserting these expressions into Eq. (98) and interchanging the integrations and the summation, the following expression is obtained

$$g_{1\ \tau=2}^{+}(x) = \int_0^1 dy G_1^{\pm}(y)\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn\ y^n x^{-(n+1)}(n+1)^2\sum_{j=0}^{\infty}\left(\frac{M^2y^2}{Q^2}\right)^j\frac{(n+j+1)!}{j!(n+1)!}\ . \tag{102}$$

The infinite sum over j in this equation is performed by observing that

$$\frac{1}{(1-r)^{n+1}} = \sum_{j=0}^{\infty}\frac{(n+j)!}{j!n!}r^j = \sum_{l,k\geq 0}\left[\begin{array}{c} k \\ l \end{array}\right](n+1)^l\frac{r^k}{k!}\ . \tag{103}$$

This relation can be derived by differentiating the generating functional of the geometric series, $(1-r)^{-1}$. Here, $\left[\begin{array}{c} k \\ l \end{array}\right]$ denotes the Stirling numbers of the first kind, cf. [22].

The integration variable n is brought into exponential from by introducing differential operators w.r.t. x

$$g_{1\ \tau=2}^{\pm}(x) = x\frac{d}{dx}x\frac{d}{dx}\int_0^1 dy\frac{G_1^{\pm}(y)}{x\left(1-M^2y^2/Q^2\right)}\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn y^n x^{-n}\left(1-\frac{M^2y^2}{Q^2}\right)^{-n}. \tag{104}$$

The n–integration leads to a δ-function

$$g^{\pm}_{1\ \tau=2}(x) = x\frac{d}{dx}x\frac{d}{dx}\int_0^1 dy\frac{G_1^{\pm}(y)}{x\left(1-\frac{M^2y^2}{Q^2}\right)}\delta\left[\ln(y)-\ln(x)-\ln\left(1-\frac{M^2y^2}{Q^2}\right)\right] . \quad (105)$$

Finally one obtains

$$g^{\pm}_{1\ \tau=2}(x) = x\frac{d}{dx}x\frac{d}{dx}\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_1^{\pm}(\xi)}{\xi}\right] . \quad (106)$$

Here, ξ denotes the Nachtmann–variable [10],

$$\xi = \frac{2x}{1+(1+4M^2x^2/Q^2)^{1/2}} . \quad (107)$$

Similar expressions are obtained for the other structure functions

$$g^{\pm}_{2\ \tau=2}(x) = -x\frac{d^2}{dx^2}x\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_1^{\pm}(\xi)}{\xi}\right] , \quad (108)$$

$$g^{\pm}_{3\ \tau=2}(x) = 2x^2\frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_2^{\pm}(\xi)}{\xi^2}\right] , \quad (109)$$

$$g^{\pm}_{4\ \tau=2}(x) = -x^2\frac{d}{dx}x\frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_2^{\pm}(\xi)}{\xi^2}\right] , \quad (110)$$

$$g^{\pm}_{5\ \tau=2}(x) = -x\frac{d}{dx}\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_3^{\pm}(\xi)}{\xi}\right]+\frac{M^2}{Q^2}x^2\frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_2^{\pm}(\xi)}{\xi}\right] . \quad (111)$$

The functions $G_2^{\pm}(y)$ and $G_3^{\pm}(y)$ are related to the distribution function $F^{\pm q}(y)$ by

$$G_2^{\pm}(y) = \sum_q g_V^q g_A^q \int_y^1 dy_1 \int_{y_1}^1 \frac{dy_2}{y_2}\int_{y_2}^1 \frac{dy_3}{y_3}F^{\pm q}(y_3) , \quad (112)$$

$$G_3^{\pm}(y) = \sum_q \frac{1}{2}\, g_V^q g_A^q \int_y^1 \frac{dy_1}{y_1}F^{\pm q}(y_1) . \quad (113)$$

Performing the derivatives in Eqs. (106, 108–111), the twist–2 contributions to the structure functions can be expressed in terms of the distribution functions $F^{\pm q}(\xi)$ as follows :

$$\begin{aligned} g^{\pm}_{1\ \tau=2}(x) &= \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4}\Bigg\{\frac{x}{\xi}\frac{F^{\pm q}(\xi)}{(1+4M^2x^2/Q^2)^{3/2}} \\ &+ \frac{4M^2x^2}{Q^2}\frac{(x+\xi)}{\xi(1+4M^2x^2/Q^2)^2}\int_\xi^1\frac{d\xi_1}{\xi_1}F^{\pm q}(\xi_1) \\ &- \frac{4M^2x^2}{Q^2}\frac{(2-4M^2x^2/Q^2)}{2(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1\frac{d\xi_1}{\xi_1}\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}F^{\pm q}(\xi_2)\Bigg\} , \quad (114) \\ g^{\pm}_{2\ \tau=2}(x) &= \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4}\Bigg\{-\frac{x}{\xi}\frac{F^{\pm q}(\xi)}{(1+4M^2x^2/Q^2)^{3/2}} \\ &+ \frac{x(1-4M^2x\xi/Q^2)}{\xi(1+4M^2x^2/Q^2)^2}\int_\xi^1\frac{d\xi_1}{\xi_1}F^{\pm q}(\xi_1) \end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{3}{2}\frac{4M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1 \frac{d\xi_1}{\xi_1}\int_{\xi_1}^1 \frac{d\xi_2}{\xi_2}F^{\pm q}(\xi_2)\Bigg\} \;, \qquad (115)\\
g^{\pm}_{3\ \tau=2}(x) &= \sum_q g_V^q g_A^q\Bigg\{\frac{2x^2}{\xi(1+4M^2x^2/Q^2)^{3/2}}\int_\xi^1 \frac{d\xi_1}{\xi_1}F^{\pm q}(\xi_1)\\
&+ \quad \frac{12M^2x^3/Q^2}{(1+4M^2x^2/Q^2)^2}\int_\xi^1 \frac{d\xi_1}{\xi_1}\int_{\xi_1}^1 \frac{d\xi_2}{\xi_2}F^{\pm q}(\xi_2)\\
&+ \quad \frac{3}{2}\frac{(4M^2x^2/Q^2)^2}{(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1 d\xi_1\int_{\xi_1}^1 \frac{d\xi_2}{\xi_2}\int_{\xi_2}^1 \frac{d\xi_3}{\xi_3}F^{\pm q}(\xi_3)\Bigg\} \;, \qquad (116)\\
g^{\pm}_{4\ \tau=2}(x) &= \sum_q g_V^q g_A^q\Bigg\{\frac{x^2}{\xi}\frac{F^{\pm q}(\xi)}{(1+4M^2x^2/Q^2)^2}+\frac{8M^2x^2}{Q^2}\frac{x(x+\xi)}{\xi(1+4M^2x^2/Q^2)^3}\int_\xi^1 \frac{d\xi_1}{\xi_1}F^{\pm q}(\xi_1)\\
&- \quad 3\frac{4M^2x^3}{Q^2}\frac{(1-M^2x(4x+\xi)/Q^2)}{(1+4M^2x^2/Q^2)^3}\int_\xi^1 \frac{d\xi_1}{\xi_1}\int_{\xi_1}^1 \frac{d\xi_2}{\xi_2}F^{\pm q}(\xi_2)\\
&- \quad \frac{3}{4}\frac{(4M^2x^2/Q^2)^2(3-8M^2x^2/Q^2)}{(1+4M^2x^2/Q^2)^{7/2}}\int_\xi^1 d\xi_1\int_{\xi_1}^1 \frac{d\xi_2}{\xi_2}\int_{\xi_2}^1 \frac{d\xi_3}{\xi_3}F^{\pm q}(\xi_3)\Bigg\} \;, \qquad (117)\\
g^{\pm}_{5\ \tau=2}(x) &= \sum_q \frac{1}{2}\ g_V^q g_A^q\Bigg\{\frac{x}{\xi(1+4M^2x^2/Q^2)}F^{\pm q}(\xi)\\
&+ \quad 2\frac{M^2x^2}{Q^2}\Bigg(\frac{(1+\xi)}{\xi(1+4M^2x^2/Q^2)^{3/2}}\int_\xi^1 \frac{d\xi_1}{\xi_1}F^{\pm q}(\xi_1)\\
&+ \quad \frac{2(x-2\xi)}{\xi(1+4M^2x^2/Q^2)^2}\int_\xi^1 \frac{d\xi_1}{\xi_1}\int_{\xi_1}^1 \frac{d\xi_2}{\xi_2}F^{\pm q}(\xi_2)\\
&- \quad \frac{6M^2x/Q^2}{(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1 d\xi_1\int_{\xi_1}^1 \frac{d\xi_2}{\xi_2}\int_{\xi_2}^1 \frac{d\xi_3}{\xi_3}F^{\pm q}(\xi_3)\Bigg)\Bigg\} \;. \qquad (118)
\end{aligned}
$$

In the same way the moments of the corresponding twist–3 contributions, Eqs. (88–97), are inverted. One obtains the following x-space expressions :

$$
\begin{aligned}
g^{\pm}_{1\ \tau=3}(x,Q^2) &= \frac{4M^2}{Q^2}x^2\frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}H_1^{\pm}(\xi)\right] \;, \qquad (119)\\
g^{\pm}_{2\ \tau=3}(x,Q^2) &= x\frac{d^2}{dx^2}\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}}H_1^{\pm}(\xi)\right] \;, \qquad (120)\\
g^{\pm}_{3\ \tau=3}(x,Q^2) &= -\left(x^2\frac{d^3}{dx^3}+4\frac{M^2x^2}{Q^2}x\frac{d^2}{dx^2}x\right)\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}\frac{H_2^{\pm}(\xi)}{\xi}\right] \;, \qquad (121)\\
g^{\pm}_{4\ \tau=3}(x,Q^2) &= -4\frac{M^2}{Q^2}x^3\frac{d^3}{dx^3}\left[\frac{x^3}{(1+4M^2x^2/Q^2)^{1/2}}\frac{H_2^{\pm}(\xi)}{\xi}\right] \;, \qquad (122)\\
g^{\pm}_{5\ \tau=3}(x,Q^2) &= -2\frac{M^2}{Q^2}x^2\frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}\frac{H_2^{\pm}(\xi)}{\xi}\right] \;. \qquad (123)
\end{aligned}
$$

Here we define

$$
\begin{aligned}
H_1^{\pm}(y) &= \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4}\int_y^1 \frac{dy_1}{y_1}\int_{y_1}^1 \frac{dy_2}{y_2}D^{\pm q}(y_2) \;, \qquad (124)\\
H_2^{\pm}(y) &= \sum_q g_V^q g_A^q\int_y^1 dy_1\int_{y_1}^1 \frac{dy_2}{y_2}\int_{y_2}^1 \frac{dy_3}{y_3}D^{\pm q}(y_3) \;. \qquad (125)
\end{aligned}
$$

The matrix elements of the twist–3 operators are the moments of the distribution function $D^{\pm q}(x)$ in the massless limit,

$$d_n^{\pm q} = \int_0^1 dx x^n D^{\pm q}(x) \ , \tag{126}$$

which has, however, no partonic interpretation. Performing the derivatives in Eqs. (119–123) the following representation for the twist–3 parts of the polarized structure functions are obtained :

$$\begin{aligned}
g_{1\ \tau=3}^{\pm}(x,Q^2) &= \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4}\frac{4M^2x^2}{Q^2}\Bigg\{\frac{D^{\pm q}(\xi)}{(1+4M^2x^2/Q^2)^{3/2}} \\
&- \frac{3}{(1+4M^2x^2/Q^2)^2}\int_\xi^1 \frac{d\xi_1}{\xi_1}D^q(\xi_1) \\
&+ \frac{(2-4M^2x^2/Q^2)}{(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1\frac{d\xi_1}{\xi_1}\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}D^{\pm q}(\xi_2)\Bigg\} \ , \qquad (127) \\
g_{2\ \tau=3}^{\pm}(x,Q^2) &= \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4}\Bigg\{\frac{D^{\pm q}(\xi)}{(1+4M^2x^2/Q^2)^{3/2}} \\
&- \frac{1-8M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^2}\int_\xi^1 \frac{d\xi_1}{\xi_1}D^q(\xi_1) \\
&- \frac{12M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1\frac{d\xi_1}{\xi_1}\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}D^{\pm q}(\xi_2)\Bigg\} \ , \qquad (128) \\
g_{3\ \tau=3}^{\pm}(x,Q^2) &= \sum_q g_V^q g_A^q\Bigg\{\frac{xD^{\pm q}(\xi)}{(1+4M^2x^2/Q^2)} - \frac{2x(1-M^2x\xi/Q^2)}{(1+4M^2x^2/Q^2)^{3/2}}\int_\xi^1\frac{d\xi_1}{\xi_1}D^{\pm q}(\xi_1) \\
&- 6\frac{M^2x^2}{Q^2}\frac{(2x+\xi)}{(1+4M^2x^2/Q^2)^2}\int_\xi^1\frac{d\xi_1}{\xi_1}\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}D^{\pm q}(\xi_2) \\
&+ 6\frac{M^2x^2}{Q^2}\frac{(1-4M^2x^2/Q^2)}{(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1 d\xi_1\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}\int_{\xi_2}^1\frac{d\xi_3}{\xi_3}D^{\pm q}(\xi_3)\Bigg\} \ , \qquad (129) \\
g_{4\ \tau=3}^{\pm}(x,Q^2) &= -\sum_q g_V^q g_A^q\frac{4M^2x^2}{Q^2}\Bigg\{\frac{xD^{\pm q}(\xi)}{(1+4M^2x^2/Q^2)^2} \\
&+ \frac{x^2(5-2M^2x\xi/Q^2)}{(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1\frac{d\xi_1}{\xi_1}D^{\pm q}(\xi_1) \\
&+ \frac{6x^2(2x^2-3\xi^3)}{\xi^2(1+4M^2x^2/Q^2)^3}\int_\xi^1\frac{d\xi_1}{\xi_1}\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}D^{\pm q}(\xi_2) \\
&- 24\frac{M^2x^2}{Q^2}\frac{(1-M^2x^2/Q^2)}{(1+4M^2x^2/Q^2)^{7/2}}\int_\xi^1 d\xi_1\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}\int_{\xi_2}^1\frac{d\xi_3}{\xi_3}D^{\pm q}(\xi_3)\Bigg\} \ , \qquad (130) \\
g_{5\ \tau=3}^{\pm}(x,Q^2) &= -2\sum_q g_V^q g_A^q\frac{M^2x^2}{Q^2}\Bigg\{\frac{1}{(1+4M^2x^2/Q^2)^{3/2}}\int_\xi^1\frac{d\xi_1}{\xi_1}D^{\pm q}(\xi_1) \\
&- \frac{2(1-M^2x\xi/Q^2)}{(1+4M^2x^2/Q^2)^2}\int_\xi^1\frac{d\xi_1}{\xi_1}\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}D^{\pm q}(\xi_2) \\
&- \frac{6M^2x/Q^2}{(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1 d\xi_1\int_{\xi_1}^1\frac{d\xi_2}{\xi_2}\int_{\xi_2}^1\frac{d\xi_3}{\xi_3}D^{\pm q}(\xi_3)\Bigg\} \ . \qquad (131)
\end{aligned}$$

The multiple integrals in the above expressions may be simplified further to single integrals by

using the idendities

$$\int_{\xi}^{1}\frac{d\xi_1}{\xi_1}\int_{\xi_1}^{1}\frac{d\xi_2}{\xi_2}\phi(\xi_2) = \int_{\xi}^{1}\frac{d\xi_1}{\xi_1}\log\left(\frac{\xi_1}{\xi}\right)\phi(\xi_1)\ , \tag{132}$$

$$\int_{\xi}^{1}d\xi_1\int_{\xi_1}^{1}\frac{d\xi_2}{\xi_2}\phi(\xi_2)\int_{\xi_2}^{1}\frac{d\xi_3}{\xi_3}\phi(\xi_3) = \int_{\xi}^{1}\frac{d\xi_1}{\xi_1}\left[\xi_1-\xi-\xi\log\left(\frac{\xi_1}{\xi}\right)\right]\phi(\xi_1)\ . \tag{133}$$

We would like to comment on the behavior of Eqs. (114–118,127–131) in the range of large values of x. At first sight these relations for the structure functions are not correct. The structure functions should vanish at $x = 1$, which is not the case for the r.h.s. of Eqs. (114–118,127–131). The inconsistency of these equations can be shown also by simply resumming the (M^2/Q^2)–terms in Eqs. (78–97), see also [23]. As an example, for the moments of $g_1^+(x,Q^2)$ one obtains

$$\int_0^1 dx\,x^n g_1^+(x,Q^2) = \int_0^{1/(1-M^2/Q^2)} dx\,x^{n+1}\frac{d}{dx}x\frac{d}{dx}\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_1^+(\xi)}{\xi}\right]. \tag{134}$$

The integrand in the r.h.s. of Eq. (134) is just the expression of $g_1^+(x,Q^2)$ after the inverse Mellin transform, Eq. (114). Therefore, the x–space expression for $g_1^+(x,Q^2)$, Eq. (114), cannot be completely correct because of the different ranges of integration.

This problem can be resolved if we assume that $F(\xi)$, and consequently $G_1(\xi)$, vanishes for $\xi > \xi_{th}$, where $\xi_{th} = \xi(x = 1)$. This assumption is equivalent to the introduction of a kinematic threshold factor [11] in the moments of distribution functions, as was proposed in Ref. [24]. Moreover, only in this case both the methods of Refs. [8, 10] are consistent. The matrix elements of operators calculated by Nachtmann moments of the structure functions (114–118) coincide with the definition of the matrix elements through the moments of the function $F(x)$, Eq. (99), as was observed for the unpolarized structure functions in Ref. [25]. In the range of large values of x the contributions due to dynamical higher twist operators are increasingly important. As well–known the number of higher twist operators grows with the spin index n the more the larger the twist τ [3] and contribute significantly because of their number as $x \to 1$. Therefore the moments of the distribution function $F(x)$ cannot be expressed by the matrix elements of the lowest twist operators only[4]. Conversely, the matrix element of the operators in the expressions of the moments, Eqs. (78–87), cannot be approximated by Eq. (99) alone. As a consequence it turns out, that in the approximation considered the results for the structure functions in x-space are reliable only at $\xi \ll \xi_{th}$.

After having performed the resummation of the M^2/Q^2 effects in Eqs. (114–118,127–131) one may try a perturbative treatment if $M^2/Q^2 \ll 1$ by expanding these expressions into Taylor series. Let us consider the principle relations for the structure function $g_1(x,Q^2)$ as an example. Its moments are given by

$$\mathcal{M}\left[g_1\right](N,Q^2) = \int_0^1 dx x^n g_1(x,Q^2)\ . \tag{135}$$

[3] The number of these operators $N(n,\tau)$ is a complicated function in most field theories, cf. e.g. [5, 7].

[4] Quite a variety of empirical ansätze were tried in the literature to model higher twist contributions to structure functions, see [26, 27] and references therein. They are strongly correlated to the parameters of the lowest twist distributions [27]. These terms are not derived in QCD. Due to the structure of higher twist operators and coefficient functions, cf. Refs. [5–7], it is expected that the structure of the higher twist contributions is even more complicated than assumed in the ansätze quoted above.

The first terms of the corresponding Taylor series are

$$\begin{aligned}
g_1^n &= g_{10}^n + \frac{M^2}{Q^2}\left\{\frac{(n+1)(n+2)(n+5)}{(n+3)^2}g_{10}^{n+2} + 4\frac{n+1}{n+3}g_{20}^{n+2}\right\} \\
&+ \left(\frac{M^2}{Q^2}\right)^2\left\{\frac{(n+1)(n+2)(n+3)(n+9)}{2(n+5)^2}g_{10}^{n+4} + 4\frac{(n+1)(n+2)(n+3)}{(n+4)(n+5)}g_{20}^{n+4}\right\} \\
&+ \left(\frac{M^2}{Q^2}\right)^3\left\{\frac{(n+1)(n+2)(n+3)(n+4)(n+13)}{6(n+7)^2}g_{10}^{n+6}\right. \\
&\qquad \left. +4\frac{(n+1)(n+2)(n+3)(n+4)}{2(n+6)(n+7)}g_{20}^{n+6}\right\} + \mathcal{O}\left(\frac{M^8}{Q^8}\right), \qquad (136)
\end{aligned}$$

Here g_{10}^n and g_{20}^n are the moments of the corresponding structure functions in the limit $M \to 0$, with

$$\begin{aligned}
g_{10}^n &= a_n \,, \\
g_{20}^n &= \frac{n}{n+1}(d_n - a_n) \,. \qquad (137)
\end{aligned}$$

Let us consider furthermore the twist–2 part of $g_1(x)$ only,[5]

$$\begin{aligned}
g_1^n &= g_{10}^n + \frac{M^2}{Q^2}\left\{\frac{(n+1)^2(n+2)}{(n+3)^2}g_{10}^{n+2}\right\} \\
&+ \left(\frac{M^2}{Q^2}\right)^2\left\{\frac{(n+1)^2(n+2)(n+3)}{2(n+5)^2}g_{10}^{n+4}\right\} \\
&+ \left(\frac{M^2}{Q^2}\right)^3\left\{\frac{(n+1)^2(n+2)(n+3)(n+4)}{6(n+7)^2}g_{10}^{n+6}\right\} + \mathcal{O}\left(\frac{M^8}{Q^8}\right) \,. \qquad (138)
\end{aligned}$$

The inverse Mellin transforms of the prefactors of g_{10}^{n+2} and g_{10}^{n+4} in Eq. (138) are

$$\begin{aligned}
\mathcal{M}^{-1}\left[\frac{(n+1)^2(n+2)}{(n+3)^2}\right](x) &= -\left(x^3\frac{\mathrm{d}}{\mathrm{d}x} + 5x^2\right)\delta(1-x) + 4x^2\ln x + 8x^2 \,, \qquad (139) \\
\mathcal{M}^{-1}\left[\frac{(n+1)^2(n+2)(n+3)}{(n+5)^2}\right](x) &= \left(x^5\frac{\mathrm{d}^2}{\mathrm{d}x^2} + 12x^5\frac{\mathrm{d}}{\mathrm{d}x} + 64x^4\right)\delta(1-x) \\
&\qquad -96x^4\ln x - 128x^4 \,, \text{ etc.} \qquad (140)
\end{aligned}$$

One observes that the higher the exponent k of $(M^2/Q^2)^k$ the higher the order of the derivatives of the δ–function in the inverse Mellin transforms given above. Let us assume that the large x behavior of $g_{10}(x)$ is typically being described by

$$g_{10}(x) \propto (1-x)^a \,, \qquad (141)$$

with $a > 0$. The target mass effects, if represented in terms of a Taylor expansion, yield

$$\begin{aligned}
g_1(x,Q^2) &\simeq (1-x)^a + \frac{M^2}{Q^2}\left\{ax(1-x)^{a-1} - 5(1-x)^a + I_1(x)\right\} \qquad (142) \\
&+ \frac{M^4}{Q^4}\left\{\frac{1}{2}a(a-1)x^2(1-x)^{a-2} - 7ax(1-x)^{a-1} + 31(1-x)^a + I_2(x)\right\} + \dots \,,
\end{aligned}$$

[5]The subsequent arguments are similar in the case of the twist–3 contributions.

The integrals

$$
\begin{aligned}
I_1(x) &= 4\int_x^1 \frac{dx_1}{x_1}\left[2+\ln(x_1)\right]\left(1-\frac{x}{x_1}\right)^a ,\\
I_2(x) &= -16\int_x^1 \frac{dx_1}{x_1}\left[4+3\ln(x_1)\right]\left(1-\frac{x}{x_1}\right)^a
\end{aligned} \tag{143}
$$

are regular. However, starting at $O(K)$ with $K > a$ contributions $\propto 1/(1-x)^{|K-a|}$ emerge, which diverge as $x \rightarrow 1$. The resummed expressions, on the other hand, are convergent.

8 Relations between the Structure Functions : Twist 2

In previous analyses three independent relations between the twist–2 parts of the polarized structure functions were derived in the lowest order of the coupling constant, the Dicus–relation [16], the Wandzura–Wilczek relation [17], and a third relation given in Ref. [15]. We investigate in the following the impact of the target mass corrections on these relations.

The expressions for the twist–2 contributions to the structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$, Eqs. (106, 108), can be rewritten in the following form

$$
g_1(x,Q^2) = x\frac{d}{dx}\mathcal{F}(x,Q^2) + x^2\frac{d^2}{dx^2}\mathcal{F}(x,Q^2) , \tag{144}
$$

$$
g_2(x,Q^2) = -2x\frac{d}{dx}\mathcal{F}(x,Q^2) - x^2\frac{d^2}{dx^2}\mathcal{F}(x,Q^2) . \tag{145}
$$

Here $\mathcal{F}(x,Q^2)$ is given by

$$
\mathcal{F}(x,Q^2) = \frac{x}{\sqrt{1+\dfrac{4M^2x^2}{Q^2}}}\frac{G_1(\xi)}{\xi} . \tag{146}
$$

From Eqs. (144, 145) one obtains

$$
g_2(x,Q^2) = -g_1(x,Q^2) - x\frac{d}{dx}\mathcal{F}(x,Q^2) , \tag{147}
$$

and

$$
x\frac{d}{dx}\mathcal{F}(x,Q^2) = -\int_x^1 \frac{dy}{y} g_1(y,Q^2) , \tag{148}
$$

from which the Wandzura–Wilczek relation [17]

$$
g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2) \tag{149}
$$

results. Therefore the Wandzura–Wilczek relation holds in the presence of target mass corrections. This result was obtained in Ref. [9] before. The Q^2–behavior in Eq. (149) refers to the (M^2/Q^2)–corrections in all orders. From Eqs. (78, 79, 83, 84) one obtains

$$
n\int_0^1 dx x^n g_1^i(x,Q^2) = -(n+1)\int_0^1 dx x^n g_2^i(x,Q^2), \tag{150}
$$

where $n = 2, 4...$ for $i = NC, +$ and $n = 1, 3...$ for $i = -$. Note that the moments in Eq. (150) are Cornwall–Norton moments, [28]. Moreover, it can be shown that the Wandzura–Wilczek relation is also valid for the quarkonic operators even in the massive quark case, see appendix.

The expression for $g_{2,\tau=2}(x, Q^2)$, Eq. (150), is formally consistent with the Burkhardt–Cottingham sum rule [29] in the presence of target mass corrections

$$\int_0^1 dx g_{2,\tau=2}^i(x, Q^2) = 0 \ , \tag{151}$$

although the $0th$ moment of the structure functions $g_2^i(x, Q^2)$ is not described by the local operator product expansion. The same holds for the $0th$ moment of the twist–3 contribution to $g_2^i(x, Q^2)$, cf. Eqs. (89,94).

The relation between the twist–2 contribution to the structure functions $g_3(x)$ and $g_4(x)$ obtained in Ref. [15] in the massless limit is also valid at any order of M^2/Q^2. To show this we integrate Eq. (110) which results into

$$g_3^i(x, Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4^i(y, Q^2) \ . \tag{152}$$

This relation can also be derived directly from the relation between the moments of the twist–2 parts of the structure functions $g_3(x)$ and $g_4(x)$

$$\int_0^1 dx x^n g_4^i(x, Q^2) = \frac{n+2}{2} \int_0^1 dx x^n g_3^i(x, Q^2) \ , \tag{153}$$

where $n = 2, 4...$ for $i = NC, +$ and $n = 1, 3...$ for the $i = -$.

In the limit $M \to 0$ Eqs. (81, 82, 86, 87) result into the Dicus–relation

$$\int_0^1 dx x^n g_4^i(x, Q^2) = 2 \int_0^1 dx x^{n+1} g_5^i(x, Q^2) \ , \tag{154}$$

cf. [16], where $n = 2, 4...$ for $i = NC, +$ and $n = 1, 3...$ for $i = -$, which reads in x–space

$$g_4^i(x, Q^2) = 2x g_5^i(x, Q^2) \ . \tag{155}$$

This relation is not preserved in the presence of target mass corrections as in the corresponding case for unpolarized deep inelastic scattering also the Callan–Gross relation [30]. This is expected since the tensor–structure in $W_{\mu\nu}$ in the case of g_5 and g_4 is the same as that of F_1 and F_2 for the unpolarized part except the factor of $S \cdot P$. The breaking term $\Delta_D(x, M^2/Q^2)$,

$$g_4^i(x, Q^2) = 2x g_5^i(x, Q^2) + \Delta_D\left(x, \frac{M^2}{Q^2}\right) \ , \tag{156}$$

is of $O(M^2/Q^2)$ and reads

$$\begin{aligned}
\Delta_D\left(x, \frac{M^2}{Q^2}\right) \quad = \quad & -x^2 \frac{d}{dx} x \frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}} \frac{G_2(\xi)}{\xi^2}\right] \\
& -2\frac{M^2}{Q^2} x^3 \frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}} \frac{G_2(\xi)}{\xi}\right] \\
& +2x^2 \frac{d}{dx}\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}} \frac{G_3(\xi)}{\xi}\right] \ .
\end{aligned} \tag{157}$$

9 Relations between the Structure Functions : Twist 3

9.1 General Relations

In the presence of target mass corrections all structure functions $g_i|_{i=1}^5$ contain twist–3 contributions. On the contrary, in the massless limit this is the case for the structure functions g_2 and g_3 only [15]. The structure functions are defined by the hadronic tensor $W_{\mu\nu}$, Eq. (8). From its structure alone it cannot be concluded which structure functions vanish in the limit $M \rightarrow 0$. The mass dependence of the scattering cross sections for longitudinal nucleon polarization, Eq. (10), however, reveals that both the structure functions g_2 and g_3 do only contribute at $O(M^2/Q^2)$. Therefore, a complete account for twist–3 contributions requires to consider also the nucleon mass corrections for the other polarized structure functions.

Let us consider now the relations between the twist–3 parts of the structure functions. From Eqs. (119–123) one derives the following *new* relations between twist–3 parts of different spin–dependent structure functions :

$$g^i_{1,\ \tau=3}(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g^i_{2,\ \tau=3}(x,Q^2) - 2\int_x^1 \frac{dy}{y} g^i_{2,\ \tau=3}(y,Q^2)\right] , \tag{158}$$

$$\frac{4M^2x^2}{Q^2} g^i_{3,\ \tau=3}(x,Q^2) = g^i_{4,\ \tau=3}(x,Q^2)\left(1 + \frac{4M^2x^2}{Q^2}\right) + 3\int_x^1 \frac{dy}{y} g^i_{4,\ \tau=3}(y,Q^2) , \tag{159}$$

$$2x g^i_{5,\ \tau=3}(x,Q^2) = -\int_x^1 \frac{dy}{y} g^i_{4,\ \tau=3}(y,Q^2) . \tag{160}$$

Here, Eq. (158) at the one side and Eqs. (159,160) on the other side correspond to different flavor combinations among the twist–3 contributions. This is similar to the case of the twist–2 terms, where the former case corresponds to the combination $\Delta q + \Delta\overline{q}$, and the latter to $\Delta q - \Delta\overline{q}$.

The corresponding relations for the Mellin moments of the twist–3 part of the structure functions can be derived from Eqs. (88–97) and read

$$\int_0^1 dx\ x^n g^i_{1,\ \tau=3}(x,Q^2) = \frac{n+1}{n+3}\int_0^1 dx\ x^n \frac{4M^2x^2}{Q^2} g^i_{2,\ \tau=3}(x,Q^2) , \tag{161}$$

$$\int_0^1 dx\ x^n \frac{4M^2x^2}{Q^2} g^i_{3,\ \tau=3}(x,Q^2) = \frac{n+4}{n+1}\int_0^1 dx\ x^n g^i_{4,\ \tau=3}(x,Q^2) + \int_0^1 dx\ x^n \frac{4M^2x^2}{Q^2} g^i_{4,\ \tau=3}(x,Q^2) , \tag{162}$$

$$\int_0^1 dx\ x^{n+1} g^i_{5,\ \tau=3}(x,Q^2) = -\frac{1}{2(n+1)}\int_0^1 dx\ x^n g^i_{4,\ \tau=3}(x,Q^2) . \tag{163}$$

The consistency of these relations is easily checked by partial integration. Eqs. (158–160) show that the twist–3 contributions to g_1, g_4 and g_5 vanish in the limit $M \rightarrow 0$. On the other hand, if one keeps terms of $O((M^2/Q^2)\cdot g_{2(3)})$ and twist–3 in the scattering cross sections, one has to account also for the twist–3 terms in g_1, g_4 and g_5. Because of the arguments given at the end of section 7 this requires the *complete* account for target mass corrections.

9.2 Other Relations

If the nucleon mass effects are disregarded in the operator product expansion only the structure functions g_2 and g_3 receive twist–3 contributions [15]. A relation between the matrix elements

d_n^{q+}, d_n^{q-} contributing to g_2 and g_3, respectively, can be constructed by *assuming* that

$$d_n^{q-} = d_n^{q+}\Big|_{\rm val} \; . \tag{164}$$

One obtains

$$\int_0^1 dx x^n \left[4g_5^-(x,Q^2) - \frac{n+1}{x} g_3^-(x,Q^2)\right]\Big|_{\nu n - \nu p} = (n-1)\left[d_n^{u-} - d_n^{d-}\right] \; , \tag{165}$$

$$\int_0^1 dx x^n \left[n g_1^\gamma(x,Q^2) + (n+1) g_2^\gamma(x,Q^2)\right]\Big|_{\gamma p - \gamma n} = \frac{n}{12}\left[d_n^{u+} - d_n^{d+}\right]\Big|_{\rm val}, \quad n = 2,4,\ldots \tag{166}$$

Eq. (164) allows to combine these relations to

$$g_{3,\tau=3}^{\nu n - \nu p}(x,Q^2) = 12\left[x g_{2,\tau=3}(x,Q^2) - \int_x^1 dy g_{2,\tau=3}(y,Q^2)\right]^{\gamma p - \gamma n} \; . \tag{167}$$

The target mass corrections break this relation. The correction terms to Eqs. (165,166) are

$$\int_0^1 dx x^n \left[4g_5^-(x,Q^2) - \frac{n+1}{x} g_3^-(x,Q^2)\right] = -(n-1)\sum_q g_V^q g_A^q \sum_{j=0}^\infty \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!}{j!(n-1)!}\frac{d_{n+2j}^{q-}}{n+2j} \; , \tag{168}$$

$$\int_0^1 dx x^n \left[n g_1^\gamma(x,Q^2) + (n+1) g_2^\gamma(x,Q^2)\right] = \sum_q \frac{e_q^2}{4}\sum_{j=0}^\infty \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!(n+2j)}{j!(n-1)!}\frac{d_{n+2j}^{q+}}{n+2j} \; . \tag{169}$$

In general it is not possible to absorb these contributions into the respective structure functions.

A sum rule for the first moment of the valence part of $g_1(x,Q^2)$ and $g_2(x,Q^2)$

$$\int_0^1 dx x \left[g_1^V(x) + 2 g_2^V(x)\right] = 0. \tag{170}$$

was obtained in Ref. [31]. In the zero–mass limit it was shown in Ref. [15] that this sum rule is formally consistent with the operator product expansion. The valence parts $g_1^V(x)$ and $g_2^V(x)$ cannot be isolated for electromagnetic interactions from the complete structure functions. One may, however, refer to the charged current case, Eqs. (83,84), and obtains

$$\int_0^1 dx\, x (g_1^-(x,xQ^2) + 2 g_2^-(x,Q^2)) = \sum_q \frac{(g_V^q)^2 + (g_A^q)^2}{4}\sum_{j=0}^\infty \left(\frac{M^2}{Q^2}\right)^j (j+1) d_{2j+1}^{-q} \; . \tag{171}$$

The l.h.s. of Eq. (171) contains only valence quark contributions. For individual quark flavors we may write separately

$$\int_0^1 dx\, x \left[g_1^{V_q}(x,xQ^2) + 2 g_2^{V_q}(x,Q^2)\right] = \frac{e_q^2}{4}\sum_{j=0}^\infty \left(\frac{M^2}{Q^2}\right)^j (j+1) d_{2j+1}^{V_q} \; . \tag{172}$$

The matrix elements of the twist–3 operators for odd indices, d_{2j+1}, are proportional to the mass of the quark. Let us consider the matrix elements of the following bi-local operators

$$\Theta^{\beta\mu_1\cdots\mu_n} = \mathcal{S}_n \bar{q}(\gamma_\beta\gamma_5 i D_1^\mu \ldots i D^{\mu_n} - \gamma_{\mu_1}\gamma_5 i D^\beta \ldots i D^{\mu_n}) q \; , \tag{173}$$

$$O^{\beta\mu_1\cdots\mu_n} = \mathcal{S}_n \bar{q} i\gamma_5 \sigma^{\beta\mu_1} i D^{\mu_2} \ldots i D^{\mu_{2j}} q \; , \tag{174}$$

where the symbol $\mathcal{S}_n$ denotes the symmetrization of the indices $\mu_1, \ldots, \mu_n$. The nucleon matrix elements of these operators are related by

$$\langle PS|\Theta^{\beta\mu_1\cdots\mu_n}|PS\rangle = m_q \langle PS|O^{\beta\mu_1\cdots\mu_n}|PS\rangle \; . \tag{175}$$

In the massive quark case the Θ– and O–operators are not traceless and their nucleon matrix elements are defined as follows

$$\langle PS|\Theta^{\beta\mu_1\cdots\mu_{2j}}|PS\rangle = \mathcal{S}_n d_{2j+1}^{V_q}(S^\beta P^{\mu_1}\ldots P^{\mu_{2j}} - S^{\mu_1}P^\beta \ldots P^{\mu_{2j}}) + g_{ij} \text{ terms} \; , \tag{176}$$

$$\langle PS|O^{\beta\mu_1\cdots\mu_{2j}}|PS\rangle = \mathcal{S}_n \frac{a_{2j+1}}{M}(S^\beta P^{\mu_1}\ldots P^{\mu_{2j}} - S^{\mu_1}P^\beta \ldots P^{\mu_{2j}}) + g_{ij} \text{ terms} \; . \tag{177}$$

The matrix element in the r.h.s. of Eq. (177) is related to the moments of the twist–2 structure function $h_1(x)$, cf. Ref. [32] , by

$$\int_0^1 dx\, x^{n-1} h_1(x) = \int_0^1 dx\, x^{n-1}\left[h_1(x) - (-1)^{n-1}\overline{h}_1(x)\right] = a_n \; . \tag{178}$$

Due to the same symmetry properties of operators Θ and O the tensors in the r.h.s. of the Eqs. (176) and (177) are the same. Therefore, using the expressions for nuclear matrix elements of the above operators and the relation between them, Eq. (172), the first moment of $g_1(x)+2g_2(x)$, Eq. (171), can be expressed through the moments of transversity distribution

$$\int_0^1 dx\; x\,[g_1(x) + 2g_2(x)] = \frac{e_q^2}{2}\frac{m_q}{M}\sum_{j=0}^{\infty}\left(\frac{M^2}{Q^2}\right)^j (j+1)\int_0^1 dx\; x^{2j}[h_1(x) - \overline{h}_1(x)] \; . \tag{179}$$

After resummation one obtains

$$\int_0^1 dx x\,[g_1(x) + 2g_2(x)] = \frac{e_q^2}{2}\frac{m_q}{M}\int_0^1 dx \frac{h(x) - \overline{h}(x)}{\left(1 - \dfrac{M^2x^2}{Q^2}\right)^2} \; . \tag{180}$$

For $Q^2 > M^2$, which was implicitly assumed in performing the light cone expansion above, the integral in the r.h.s. of Eq. (180) is finite. Therefore the r.h.s. of Eq. (180) vanishes in the limit $m_q \to 0$. This is equivalent to $d_{2j+1}^{V_q} \to 0$ and Eq. (170) is found to be formally consistent with the the result of the operator product expansion.

10 Conclusions

We have calculated the target mass corrections for all polarized structure functions for both neutral and charged current deep inelastic scattering in the case of conserved currents. The results were obtained by using the local light cone expansion of the Compton amplitude for forward scattering. In the evaluation we used the approach of Ref. [8]. The target mass corrections imply besides the twist–2 terms twist–3 contributions for all polarized structure functions. Only after the inclusion of the target mass corrections the description of the polarized scattering cross sections can be regarded as being completed at the twist–3 level, since the contributions to g_1, g_4 and g_5 are of the same order as those to g_2 and g_3 discussed previously. The corrections were both represented in terms of the integer moments which result from the light cone expansion

and their analytic continuation and Mellin inversion to x-space. The latter representations are directly applicable in experimental analyses.

We investigated the effect of the target mass corrections on the sum rules connecting the polarized structure functions in lowest order in the coupling constant. For the twist–2 contributions both the Wandzura–Wilczek relation [17] and the relation derived in Ref. [15] are preserved, whereas the Dicus relation [16], similarly to the Callan–Gross relation [30] in the unpolarized case, receives a correction. It was also shown that the Wandzura–Wilczek relation is preserved in the presence of quark–mass corrections. A previously derived integral relation between the twist–3 valence contributions to g_2 and g_3 [15] receives target mass corrections. A relation for the first moment of a combination between g_1 and g_2 [31] is preserved.

Three new integral relations were derived for the twist–3 contributions of the polarized structure functions. They hold without further assumption on the flavor combinations of the related structure functions. In the case of the relation between the twist–3 contributions to g_1 and g_2 experimental test can be performed in the foreseeable future by precise measurements of the longitudinal and transversely polarized deep inelastic eN–scattering cross sections at lower values of Q^2.

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11 Appendix : The quark mass contribution to the Wandzura-Wilczek relation

We consider the parts of forward Compton scattering amplitude $\widehat{T}^i_{\mu\nu,spin}$, Eqs. (52) and (53), which correspond to the structure functions $g_1(x)$ and $g_2(x)$ in the massive quarks case,[6]

$$\widehat{T}_1^{\pm} \sim \varepsilon_{\mu\alpha\nu\beta} q^\alpha \sum_{n\ even,odd} q^{\mu_1}\cdots q^{\mu_n} \Big(\frac{2}{Q^2 - m_X^2 + m_q^2}\Big)^{n+1} \Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} \ . \tag{181}$$

Here m_X is the mass of the struck quark and m_q the final quark mass. The initial–state quark is not necessarily on shell, but we assume that its wave function is the usual free-particle Dirac spinor for a particle with mass m_X.

In the massive quark case the Θ–operators are not traceless anymore and have no definite twist. To construct the nucleon matrix elements for these operators, we have to decompose them into traceless ones. The most general form for a such decomposition is

$$\begin{aligned} \Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} &= \sum_{j=0}^{\infty} A(n,j) \mathcal{O}^{\pm\beta\{\mu_1\cdots\mu_{n-2j}\}} \underbrace{g\ldots g}_{j}\ (m_X^2)^j \\ &+ \sum_{j=0}^{\infty} B(n,j) \mathcal{O}^{\pm\mu_\alpha\{\beta\mu_1\cdots\mu_{n-2j-1}\}} \underbrace{g\cdots g}_{j}\ (m_X^2)^j \\ &+ \sum_{j=0}^{\infty} C(n,j) \mathcal{O}^{\pm\mu_\alpha\{\mu_1\cdots\mu_{n-2j}\}} \underbrace{g\ldots g}_{j} g_{\beta\mu_\beta}\ (m_X^2)^j \ . \end{aligned} \tag{182}$$

Because of the antisymmetric tensor $\varepsilon_{\mu\alpha\nu\beta}$ the third term $\propto C(n,j)$ does not contribute to $\widehat{T}_1^{\pm}$. Symmetrizing the other two operators in Eq. (182) we separate the twist–2 part in the forward Compton scattering amplitude. The symmetric part of the remaining two operators are identical. Therefore one obtains

$$\begin{aligned} \widehat{T}_1^{\pm} &\sim \varepsilon_{\mu\alpha\nu\beta} q^\alpha \sum_{n\ even,odd} \sum_{j=0}^{[n/2]} \left(\frac{2}{Q^2 - m_X^2 + m_q^2}\right)^{n+1} [A(n,j) + B(n,j)] \\ &\quad \times q^{\mu_1}\cdots q^{\mu_n} \Theta^{\pm\beta\{\mu_1\cdots\mu_{n-2j}\}} \underbrace{g\ldots g}_{j} (m_X^2)^j \\ &= \varepsilon_{\mu\alpha\nu\beta} q^\alpha \sum_{n\ even,odd} \sum_{j=0}^{[n/2]} \left(\frac{Q^2}{Q^2 - m_X^2 + m_q^2}\right)^{n+1} \left(\frac{2}{Q^2}\right)^{n-2j+1} \left(\frac{-4m_X^2}{Q^2}\right)^{j} \\ &\quad \times [A(n,j) + B(n,j)]\, q^{\mu_1}\cdots q^{\mu_{n-2j}} \Theta^{\pm\beta\{\mu_1\cdots\mu_{n-2j}\}} \\ &= \varepsilon_{\mu\alpha\nu\beta} q^\alpha \sum_{l\ \ even,odd} \left(\frac{2}{Q^2 - m_X^2 + m_q^2}\right)^{l+1} X_l \left[\frac{4m_X^2 Q^2}{(Q^2 - m_X^2 + m_q^2)^2}\right] \\ &\quad \times q^{\mu_1}\cdots q^{\mu_l} \Theta^{\pm\beta\{\mu_1\cdots\mu_l\}} \ . \end{aligned} \tag{183}$$

In the last step we changed the summation index from n to $l = n - 2j$ and introduced the function $X_l(z)$,

$$X_l(z) = \sum_{j=0} (-z)^j \left[A(l+2j,j) + B(l+2j,j)\right] \ . \tag{184}$$

[6] The resummation of quark mass effects was considered for the unpolarized structure functions in Refs. [8, 33].

As can be seen from Eq. (183) we obtain the same tensor structure as in Eq. (65) which leads to the Wandzura–Wilczek relation. Because the dependence on the quark masses is the same in the structure functions $g_1(x)$ and $g_2(x)$ the Wandzura–Wilczek relation is not violated by these corrections.

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